13.1 GENERAL: Every seismically isolated structure and every portion thereof shall be designed and constructed in accordance with the requirements of this section and the applicable requirements of Chapter 1. The lateral-force-resisting system and the isolation system shall be designed to resist the deformations and stresses produced by the effects of seismic ground motions as provided in this section.

13.2 CRITERIA SELECTION:

13.2.1 Basis for Design: The procedures and limitations for the design of seismically isolated structures shall be determined considering zoning, site characteristics, vertical acceleration, cracked section properties of concrete and masonry members, Seismic Use Group, configuration, structural system, and height in accordance with Sec. 5.2 except as noted below.

13.2.2 Stability of the Isolation System: The stability of the vertical load-carrying elements of the isolation system shall be verified by analysis and test, as required, for lateral seismic displacement equal to the total maximum displacement.

13.2.3 Seismic Use Group: All portions of the structure, including the structure above the isolation system, shall be assigned a Seismic Use Group in accordance with the requirements of Sec. 1.3. The Occupancy Importance Factor shall be taken as 1.0 for a seismically isolated structure, regardless of its Seismic Use Group categorization. The Component Importance Factor shall be selected in accordance with Sec. 6.1.5.

13.2.4 Configuration Requirements: Each structure shall be designated as being regular or irregular on the basis of the structural configuration above the isolation system in accordance with the requirements of Sec. 5.2.

13.2.5 Selection of Lateral Response Procedure:

13.2.5.1 General: Any seismically isolated structure is permitted to be and certain seismically isolated structures defined below shall be designed using the dynamic lateral response procedure of Sec. 13.4.

13.2.5.2 Equivalent Lateral Force Procedure: The equivalent-lateral-response procedure of Sec. 13.3 is permitted to be used for design of a seismically isolated structure provided that:

1. The structure is located at a site with $S_1$ less than or equal to 0.60;
2. The structure is located on a Class A, B, C, or D site;
3. The structure above the isolation interface is not more than four stories or 65 ft (20 m) in height;
4. The effective period of the isolated structure, $T_M$, is less than or equal to 3.0 sec.
5. The effective period of the isolated structure, $T_D$, is greater than three times the elastic, fixed-base period of the structure above the isolation system as determined by Eq. 5.4.2.1-1 or 5.4.2.1-2;

6. The structure above the isolation system is of regular configuration; and

7. The isolation system meets all of the following criteria:
   a. The effective stiffness of the isolation system at the design displacement is greater than one third of the effective stiffness at 20 percent of the design displacement,
   b. The isolation system is capable of producing a restoring force as specified in Sec. 13.6.2.4,
   c. The isolation system has force-deflection properties that are independent of the rate of loading,
   d. The isolation system has force-deflection properties that are independent of vertical load and bilateral load, and
   e. The isolation system does not limit maximum capable earthquake displacement to less than $S_{mf}/S_{D1}$ times the total design displacement.

13.2.5.3 Dynamic Analysis: A dynamic analysis is permitted to be used for the design of any structure but shall be used for the design of all isolated structures not satisfying Sec. 13.2.5.2. The dynamic lateral response procedure of Sec. 13.4 shall be used for design of seismically isolated structures as specified below.

13.2.5.3.1 Response-Spectrum Analysis: Response-spectrum analysis is permitted to be used for design of a seismically isolated structure provided that:

1. The structure is located on a Class A, B, C, or D site and
2. The isolation system meets the criteria of Item 7 of Sec. 13.2.5.2.

13.2.5.3.2 Time-History Analysis: Time-history analysis is permitted to be used for design of any seismically isolated structure and shall be used for design of all seismically isolated structures not meeting the criteria of Sec. 13.2.5.3.1.

13.2.5.3.3 Site-Specific Design Spectra: Site-specific ground-motion spectra of the design earthquake and the maximum considered earthquake developed in accordance with Sec. 13.4.4.1 shall be used for design and analysis of all seismically isolated structures if any one of the following conditions apply:

1. The structure is located on a Class F site or
2. The structure is located at a site with $S_I$ greater than 0.60.
13.3 EQUIVALENT LATERAL FORCE PROCEDURE:

13.3.1 General: Except as provided in Sec. 13.4, every seismically isolated structure or portion thereof shall be designed and constructed to resist minimum earthquake displacements and forces as specified by this section and the applicable requirements of Sec. 5.4.

13.3.2 Deformation Characteristics of the Isolation System: Minimum lateral earthquake design displacement and forces on seismically isolated structures shall be based on the deformation characteristics of the isolation system. The deformation characteristics of the isolation system shall explicitly include the effects of the wind-restraint system if such a system is used to meet the design requirements of these Provisions. The deformation characteristics of the isolation system shall be based on properly substantiated tests performed in accordance with Sec. 13.9.

13.3.3 Minimum Lateral Displacements:

13.3.3.1 Design Displacement: The isolation system shall be designed and constructed to withstand minimum lateral earthquake displacements that act in the direction of each of the main horizontal axes of the structure in accordance with the following:

\[
D_D = \left( \frac{g}{4\pi^2} \right) \frac{S_{D1}T_D}{B_D} \tag{13.3.3.1}
\]

where:

- \( g \) = acceleration of gravity. The units of the acceleration of gravity, \( g \), are in./sec\(^2\) (mm/sec\(^2\)) if the units of the design displacement, \( D_D \), are inches (mm);
- \( S_{D1} \) = design 5 percent damped spectral acceleration at 1 sec period as determined in Sec. 4.1.1;
- \( T_D \) = effective period, in seconds (sec), of seismically isolated structure at the design displacement in the direction under consideration, as prescribed by Eq. 13.3.3.2; and
- \( B_D \) = numerical coefficient related to the effective damping of the isolation system at the design displacement, \( \beta_D \), as set forth in Table 13.3.3.1.

<table>
<thead>
<tr>
<th>Effective Damping, ( \beta_D ) or ( \beta_M ) (Percentage of Critical)(^ab)</th>
<th>( B_D ) or ( B_M ) Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \leq 2% )</td>
<td>0.8</td>
</tr>
<tr>
<td>5%</td>
<td>1.0</td>
</tr>
<tr>
<td>10%</td>
<td>1.2</td>
</tr>
<tr>
<td>20%</td>
<td>1.5</td>
</tr>
<tr>
<td>30%</td>
<td>1.7</td>
</tr>
<tr>
<td>40%</td>
<td>1.9</td>
</tr>
<tr>
<td>( \geq 50% )</td>
<td>2.0</td>
</tr>
</tbody>
</table>
NOTES for Table 13.3.3.1

\[ a \] The damping coefficient shall be based on the effective damping of the isolation system determined in accordance with the requirements of Sec. 13.9.5.2.

\[ b \] The damping coefficient shall be based on linear interpolation for effective damping values other than those given.

13.3.3.2 Effective Period: The effective period of the isolated structure, \( T_D \), shall be determined using the deformational characteristics of the isolation system in accordance with the following equation:

\[
T_D = 2\pi \sqrt{\frac{W}{k_{D_{\text{min}}}g}}
\]  

(13.3.3.2)

where:

\( W \) = total seismic dead load weight of the structure above the isolation interface as defined in Sec. 5.4.1 and 5.5.3 (kip or kN);

\( k_{D_{\text{min}}} \) = minimum effective stiffness, in kips/inch (kN/mm), of the isolation system at the design displacement in the horizontal direction under consideration as prescribed by Eq. 13.9.5.1-2; and

\( g \) = acceleration of gravity. The units of the acceleration of gravity, \( g \), are in./sec\(^2\) (mm/sec\(^2\)) if the units of the design displacement, \( D_D \), are inches (mm).

13.3.3.3 Maximum Displacement: The maximum displacement of the isolation system, \( D_M \), in the most critical direction of horizontal response shall be calculated in accordance with the formula:

\[
D_M = \left( \frac{g}{4\pi^2} \right) S_{M1} T_M \frac{R}{\bar{R}}
\]  

(13.3.3.3)

where:

\( g \) = acceleration of gravity. The units of the acceleration of gravity, \( g \), are in./sec\(^2\) (mm/sec\(^2\)) if the units of the design displacement, \( D_D \), are inches (mm);

\( S_{M1} \) = maximum considered 5 percent damped spectral acceleration at 1 sec period as determined in Sec. 4.1.1;
\[ T_M = \text{effective period, in seconds (sec), of seismic-isolated structure at the maximum displacement in the direction under consideration as prescribed by Eq. 13.3.3.4; and} \]

\[ B_M = \text{numerical coefficient related to the effective damping of the isolation system at the maximum displacement, } \beta_D, \text{ as set forth in Table 13.3.3.1.} \]

13.3.3.4 Effective Period at Maximum Displacement: The effective period of the isolated structure at maximum displacement, \( T_M \), shall be determined using the deformational characteristics of the isolation system in accordance with the equation:

\[
T_M = 2\pi \sqrt{\frac{W}{k_{M\text{min}}g}} 
\]

(13.3.3.4)

where:

\[ W = \text{total seismic dead load weight of the structure above the isolation interface as defined in Sec. 5.3.2 and 5.5.3 (kip or kN);} \]

\[ k_{M\text{min}} = \text{minimum effective stiffness, in kips/inch (kN/mm), of the isolation system at the maximum displacement in the horizontal direction under consideration as prescribed by Eq. 13.9.5.1-4; and} \]

\[ g = \text{the acceleration due to gravity. The units of the acceleration of gravity, } g, \text{ are in./sec}^2 \text{ (mm/sec}^2 \text{) if the units of the design displacement, } D_D, \text{ are inches (mm).} \]

13.3.3.5 Total Displacement: The total design displacement, \( D_{TD} \), and the total maximum displacement, \( D_{TM} \), of elements of the isolation system shall include additional displacement due to actual and accidental torsion calculated considering the spatial distribution of the lateral stiffness of the isolation system and the most disadvantageous location of mass eccentricity.

The total design displacement, \( D_{TD} \), and the total maximum displacement, \( D_{TM} \), of elements of an isolation system with uniform spatial distribution of lateral stiffness shall not be taken as less than that prescribed by the following equations:

\[
D_{TD} = D_D \left[ 1 + y\left(\frac{12e}{b^2 + d^2}\right) \right] 
\]

(13.3.3.5-1)

\[
D_{TM} = D_M \left[ 1 + y\left(\frac{12e}{b^2 + d^2}\right) \right] 
\]

(13.3.3.5-2)

where:

\[ D_D = \text{design displacement, in inches (mm), at the center of rigidity of the isolation system in the direction under consideration as prescribed by Eq. 13.3.3.1;} \]
\[ D_M = \text{maximum displacement, in inches (mm), at the center of rigidity of the isolation system in the direction under consideration as prescribed in Eq. 13.3.3.3;} \]

\[ y = \text{the distance, in feet (mm), between the center of rigidity of the isolation system rigidity and the element of interest measured perpendicular to the direction of seismic loading under consideration;} \]

\[ e = \text{the actual eccentricity, in feet (mm), measured in plan between the center of mass of the structure above the isolation interface and the center of rigidity of the isolation system, plus accidental eccentricity, in feet (mm), taken as 5 percent of the longest plan dimension of the structure perpendicular to the direction of force under consideration,} \]

\[ b = \text{the shortest plan dimension of the structure, in feet (mm), measured perpendicular to } d, \text{ and} \]

\[ d = \text{the longest plan dimension of the structure, in feet (mm).} \]

The total design displacement, \(D_{td}\), and the total maximum displacement, \(D_{tm}\), is permitted to be taken as less than the value prescribed by Eq. 13.3.3.5-1 and Eq. 13.3.3.5-2, respectively, but not less than 1.1 times \(D_D\) and \(D_M\), respectively, provided the isolation system is shown by calculation to be configured to resist torsion accordingly.

### 13.3.4 Minimum Lateral Forces:

#### 13.3.4.1 Isolation System and Structural Elements At or Below the Isolation System:

The isolation system, the foundation, and all structural elements below the isolation system shall be designed and constructed to withstand a minimum lateral seismic force, \(V_b\), using all of the appropriate provisions for a nonisolated structure where:

\[ V_b = k_{D_{\text{max}}}D_D \quad (13.3.4.1) \]

where:

\[ k_{D_{\text{max}}} = \text{maximum effective stiffness, in kips/inch (kN/mm), of the isolation system at the design displacement in the horizontal direction under consideration as prescribed by Eq. 13.9.5.1-1, and} \]

\[ D_D = \text{design displacement, in inches (mm), at the center of rigidity of the isolation system in the direction under consideration as prescribed by Eq. 13.3.3.1.} \]

In all cases, \(V_b\) shall not be taken as less than the maximum force in the isolation system at any displacement up to and including the design displacement.
13.3.4.2 Structural Elements Above the Isolation System: The structure above the isolation system shall be designed and constructed to withstand a minimum shear force, $V_s$, using all of the appropriate provisions for a nonisolated structure where:

$$V_s = \frac{k_{Dmax}D_D}{R_I}$$  \hspace{1cm} (13.3.4.2)

where:

- $k_{Dmax}$ = maximum effective stiffness, in kips/inch (kN/mm), of the isolation system at the design displacement in the horizontal direction under consideration as prescribed by Eq. 13.9.5.1-1;
- $D_D$ = design displacement, in inches (mm), at the center of rigidity of the isolation system in the direction under consideration as prescribed by Eq. 13.3.3.1; and
- $R_I$ = numerical coefficient related to the type of lateral-force-resisting system above the isolation system.

The $R_I$ factor shall be based on the type of lateral-force-resisting system used for the structure above the isolation system and shall be 3/8 of the $R$ value given in Table 5.2.2 with an upper bound value not to exceed 2.0 and a lower bound value not to be less than 1.0.

13.3.4.3 Limits on $V_s$: The value of $V_s$ shall be taken as not less than the following:

1. The lateral seismic force required by Sec. 5.3 for a fixed-base structure of the same weight, $W$, and a period equal to the isolated period, $T_D$;
2. The base shear corresponding to the factored design wind load; and
3. The lateral seismic force required to fully activate the isolation system (e.g., the yield level of a softening system, the ultimate capacity of a sacrificial wind-restraint system, or the breakaway friction level of a sliding system) factored by 1.5.

13.3.5 Vertical Distribution of Force: The total force shall be distributed over the height of the structure above the isolation interface in accordance with the following equation:

$$F_x = \frac{V_s W_x h_x}{\sum_{i=1}^{n} w_i h_i}$$  \hspace{1cm} (13.3.5)

where:

- $V_s$ = total lateral seismic design force or shear on elements above the isolation system as prescribed by Eq. 13.3.4.2;
- $W_x$ = portion of $W$ that is located at or assigned to Level $x_i$. 
At each level designated as $x$, the force, $F_x$, shall be applied over the area of the structure in accordance with the mass distribution at the level. Stresses in each structural element shall be calculated as the effect of force, $F_x$, applied at the appropriate levels above the base.

**13.3.6 Drift Limits:** The maximum interstory drift of the structure above the isolation system shall not exceed $0.015h_x$. The drift shall be calculated by Eq. 5.4.6-1 with the $C_d$ factor of the isolated structure equal to the $R_I$ factor defined in Sec. 13.3.4.2.

**13.4 DYNAMIC LATERAL RESPONSE PROCEDURE:**

**13.4.1 General:** As required by Sec. 13.2, every seismically isolated structure or portion thereof shall be designed and constructed to resist earthquake displacements and forces as specified in this section and the applicable requirements of Sec. 5.5.

**13.4.2 Isolation System and Structural Elements Below the Isolation System:** The total design displacement of the isolation system shall be taken as not less than 90 percent of $D_{TD}$ as specified by Sec. 13.3.3.5.

The total maximum displacement of the isolation system shall be taken as not less than 80 percent of $D_{TM}$ as specified by Sec. 13.3.3.5.

The design lateral shear force on the isolation system and structural elements below the isolation system shall be taken as not less than 90 percent of $V_b$ as prescribed by Eq. 13.3.4.1.

The limits of the first and second paragraphs of Sec. 13.4.2 shall be evaluated using values of $D_{TD}$ and $D_{TM}$ determined in accordance with Sec. 13.3.3 except that $D'_D$ is permitted to be used in lieu of $D_D$ and $D'_M$ is permitted to be used in lieu of $D_M$ where $D'_D$ and $D'_M$ are prescribed by the following equations:

$$D'_D = \frac{D_D}{\sqrt{1 + \left(\frac{T}{T_D}\right)^2}} \quad (13.4.2-1)$$

$$D'_M = \frac{D_M}{\sqrt{1 + \left(\frac{T}{T_M}\right)^2}} \quad (13.4.2-2)$$
where:

\[ D_D = \text{design displacement}, \text{in inches (mm), at the center of rigidity of the isolation system in the direction under consideration as prescribed by Eq. 13.3.3.1;} \]

\[ D_M = \text{maximum displacement in inches (mm), at the center of rigidity of the isolation system in the direction under consideration as prescribed by Eq. 13.3.3.3;} \]

\[ T = \text{elastic, fixed-base period of the structure above the isolation system as determined by Sec. 5.3.3;} \]

\[ T_D = \text{effective period, in seconds (sec), of the seismically isolated structure at the design displacement in the direction under consideration as prescribed by Eq. 13.3.3.2;} \]

\[ T_M = \text{effective period, in seconds (sec), of the seismically isolated structure at the maximum displacement in the direction under consideration as prescribed by Eq. 13.3.3.4.} \]

13.4.3 Structural Elements Above the Isolation System: The design lateral shear force on the structure above the isolation system, if regular in configuration, shall be taken as not less than 80 percent of \( V_s \), as prescribed by Eq. 13.3.4.2 and the limits specified by Sec. 13.3.4.3.

**Exception:** The design lateral shear force on the structure above the isolation system, if regular in configuration, is permitted to be taken as less than 80 percent, but not less than 60 percent of \( V_s \), provided time-history analysis is used for design of the structure.

The design lateral shear force on the structure above the isolation system, if irregular in configuration, shall be taken as not less than \( V_s \), as prescribed by Eq. 13.3.4.2 and the limits specified by Sec. 13.3.4.3.

**Exception:** The design lateral shear force on the structure above the isolation system, if irregular in configuration, is permitted to be taken as less than 100 percent, but not less than 80 percent of \( V_s \), provided time-history analysis is used for design of the structure.

13.4.4 Ground Motion:

13.4.4.1 Design Spectra: Properly substantiated site-specific spectra are required for the design of all structures located on a Class F site or located at a site with \( S_1 \) greater than 0.60. Structures that do not require site-specific spectra and for which site specific spectra have not been calculated shall be designed using the response spectrum shape given in Figure 4.1.2.6.

A design spectrum shall be constructed for the design earthquake. This design spectrum shall be taken as not less than the design earthquake response spectrum given in Figure 4.1.2.6.

**Exception:** If a site-specific spectrum is calculated for the design earthquake, the design spectrum is permitted to be taken as less than 100 percent but not less than 80 percent of the design earthquake response spectrum given in Figure 4.1.2.6.

A design spectrum shall be constructed for the maximum considered earthquake. This design spectrum shall be taken as not less than 1.5 times the design earthquake response spectrum given in Figure 4.1.2.6. This design spectrum shall be used to determine the total maximum displacement and overturning forces for design and testing of the isolation system.
Exception: If a site-specific spectrum is calculated for the maximum considered earthquake, the design spectrum is permitted to be taken as less than 100 percent but not less than 80 percent of 1.5 times the design earthquake response spectrum given in Figure 4.1.2.6.

13.4.4.2 Time Histories: Pairs of appropriate horizontal ground motion time history components shall be selected and scaled from not less than three recorded events. Appropriate time histories shall be based on recorded events with magnitudes, fault distances and source mechanisms that are consistent with those that control the design earthquake (or maximum considered earthquake). Where three appropriate recorded ground motion time history pairs are not available, appropriate simulated ground motion time history pairs are permitted to be used to make up the total number required. For each pair of horizontal ground-motion components, the square root sum of the squares of the 5 percent damped spectrum of the scaled, horizontal components shall be constructed. The motions shall be scaled such that the average value of the square-root-sum-of-the-squares spectra does not fall below 1.3 times the 5 percent damped spectrum of the design earthquake (or maximum considered earthquake) by more than 10 percent for periods from $0.5T_D$ seconds to $1.25T_M$ seconds.

13.4.5 Mathematical Model:

13.4.5.1 General: The mathematical models of the isolated structure including the isolation system, the lateral-force-resisting system, and other structural elements shall conform to Sec. 5.5.2 and to the requirements of Sec. 13.4.5.2 and 13.4.5.3, below.

13.4.5.2 Isolation System: The isolation system shall be modeled using deformational characteristics developed and verified by test in accordance with the requirements of Sec. 13.3.2. The isolation system shall be modeled with sufficient detail to:

1. Account for the spatial distribution of isolator units;
2. Calculate translation, in both horizontal directions, and torsion of the structure above the isolation interface considering the most disadvantageous location of mass eccentricity;
3. Assess overturning/uplift forces on individual isolator units; and
4. Account for the effects of vertical load, bilateral load, and/or the rate of loading if the force deflection properties of the isolation system are dependent on one or more of these attributes.

13.4.5.3 Isolated Building:

13.4.5.3.1 Displacement: The maximum displacement of each floor and the total design displacement and total maximum displacement across the isolation system shall be calculated using a model of the isolated structure that incorporates the force-deflection characteristics of nonlinear elements of the isolation system and the lateral-force-resisting system.

Lateral-force-resisting systems with nonlinear elements include, but are not limited to, irregular structural systems designed for a lateral force less than 100 percent and regular structural systems designed for a lateral force less than 80 percent of $V_s$ as prescribed by Eq. 13.3.4.2 and the limits specified by Sec. 13.3.4.3.
13.4.5.3.2 Forces and Displacements in Elements of the Lateral-Force-Resisting System:
Design forces and displacements in elements of the lateral-force-resisting system are permitted to be calculated using a linear elastic model of the isolated structure provided that:

1. Stiffness properties assumed for nonlinear isolation-system components are based on the maximum effective stiffness of the isolation system and

2. No elements of the lateral-force-resisting system of the structure above the isolation system are nonlinear.

13.4.6 Description of Analysis Procedures:

13.4.6.1 General: Response-spectrum and time-history analyses shall be performed in accordance with Sec. 5.4 and the requirements of this section.

13.4.6.2 Input Earthquake: The design earthquake shall be used to calculate the total design displacement of the isolation system and the lateral forces and displacements of the isolated structure. The maximum considered earthquake shall be used to calculate the total maximum displacement of the isolation system.

13.4.6.3 Response-Spectrum Analysis: Response-spectrum analysis shall be performed using a modal damping value for the fundamental mode in the direction of interest not greater than the effective damping of the isolation system or 30 percent of critical, whichever is less. Modal damping values for higher modes shall be selected consistent with those appropriate for response spectrum analysis of the structure above the isolation system with a fixed base.

Response-spectrum analysis used to determine the total design displacement and the total maximum displacement shall include simultaneous excitation of the model by 100 percent of the most critical direction of ground motion and 30 percent of the ground motion on the orthogonal axis. The maximum displacement of the isolation system shall be calculated as the vectorial sum of the two orthogonal displacements.

The design shear at any story shall not be less than the story shear obtained using Eq. 13.3.5 and a value of $V_s$ taken as that equal to the base shear obtained from the response-spectrum analysis in the direction of interest.

13.4.6.4 Time-History Analysis: Time-history analysis shall be performed with at least three appropriate pairs of horizontal time-history components as defined in Sec. 13.4.4.2.

Each pair of time histories shall be applied simultaneously to the model considering the most disadvantageous location of mass eccentricity. The maximum displacement of the isolation system shall be calculated from the vectorial sum of the two orthogonal components at each time step.

The parameter of interest shall be calculated for each time-history analysis. If three time-history analyses are performed, the maximum response of the parameter of interest shall be used for design. If seven or more time-history analyses are performed, the average value of the response parameter of interest shall be used for design.

13.4.7 Design Lateral Force:
13.4.7.1 **Isolation System and Structural Elements At or Below the Isolation System:** The isolation system, foundation, and all structural elements below the isolation system shall be designed using all of the appropriate requirements for a nonisolated structure and the forces obtained from the dynamic analysis without reduction.

13.4.7.2 **Structural Elements Above the Isolation System:** Structural elements above the isolation system shall be designed using the appropriate provisions for a nonisolated structure and the forces obtained from the dynamic analysis divided by a factor of $R_f$. The $R_f$ factor shall be based on the type of lateral-force-resisting system used for the structure above the isolation system.

13.4.7.3 **Scaling of Results:** When the factored lateral shear force on structural elements, determined using either response spectrum or time-history analysis, is less than the minimum level prescribed by Sec. 13.4.2 and 13.4.3, all response parameters, including member forces and moments, shall be adjusted proportionally upward.

13.4.7.4 **Drift Limits:** Maximum interstory drift corresponding to the design lateral force including displacement due to vertical deformation of the isolation system shall not exceed the following limits:

1. The maximum interstory drift of the structure above the isolation system calculated by response spectrum analysis shall not exceed $0.015h_{ex}$ and

2. The maximum interstory drift of the structure above the isolation system calculated by time-history analysis considering the force-deflection characteristics of nonlinear elements of the lateral-force-resisting system shall not exceed $0.020h_{ex}$.

Drift shall be calculated using Eq. 5.3.8.1 with the $C_d$ factor of the isolated structure equal to the $R_f$ factor defined in Sec. 13.3.4.2.

The secondary effects of the maximum considered earthquake lateral displacement $\Delta$ of the structure above the isolation system combined with gravity forces shall be investigated if the interstory drift ratio exceeds $0.010/R_f$.

13.5 **LATERAL LOAD ON ELEMENTS OF STRUCTURES AND NONSTRUCTURAL COMPONENTS SUPPORTED BY BUILDINGS:**

13.5.1 **General:** Parts or portions of an isolated structure, permanent nonstructural components and the attachments to them, and the attachments for permanent equipment supported by a structure shall be designed to resist seismic forces and displacements as prescribed by this section and the applicable requirements of Chapter 6.

13.5.2 **Forces and Displacements:**

13.5.2.1 **Components At or Above the Isolation Interface:** Elements of seismically isolated structures and nonstructural components, or portions thereof, that are at or above the isolation interface shall be designed to resist a total lateral seismic force equal to the maximum dynamic response of the element or component under consideration.
**Exception:** Elements of seismically isolated structures and nonstructural components or portions thereof are permitted to be designed to resist total lateral seismic force as prescribed in Sec. 5.2.6 and 6.1.3 as appropriate.

13.5.2.2 Components Crossing the Isolation Interface: Elements of seismically isolated structures and nonstructural components, or portions thereof, that cross the isolation interface shall be designed to withstand the total maximum displacement.

13.5.2.3 Components Below the Isolation Interface: Elements of seismically isolated structures and nonstructural components, or portions thereof, that are below the isolation interface shall be designed and constructed in accordance with the requirements of Sec. 5.2.

13.6 Detailed System Requirements:

13.6.1 General: The isolation system and the structural system shall comply with the material requirements of these Provisions. In addition, the isolation system shall comply with the detailed system requirements of this section and the structural system shall comply with the detailed system requirements of this section and the applicable portions of Sec. 5.2.

13.6.2 Isolation System:

13.6.2.1 Environmental Conditions: In addition to the requirements for vertical and lateral loads induced by wind and earthquake, the isolation system shall be designed with consideration given to other environmental conditions including aging effects, creep, fatigue, operating temperature, and exposure to moisture or damaging substances.

13.6.2.2 Wind Forces: Isolated structures shall resist design wind loads at all levels above the isolation interface. At the isolation interface, a wind restraint system shall be provided to limit lateral displacement in the isolation system to a value equal to that required between floors of the structure above the isolation interface.

13.6.2.3 Fire Resistance: Fire resistance rating for the isolation system shall be consistent with the requirements of columns, walls, or other such elements in the same area of the structure.

13.6.2.4 Lateral-Restoring Force: The isolation system shall be configured to produce a restoring force such that the lateral force at the total design displacement is at least 0.025W greater than the lateral force at 50 percent of the total design displacement.

    **Exception:** The isolation system need not be configured to produce a restoring force, as required above, provided the isolation system is capable of remaining stable under full vertical load and accommodating a total maximum displacement equal to the greater of either 3.0 times the total design displacement or 36SM1 in. (or 915 SM1 mm).

13.6.2.5 Displacement Restraint: The isolation system is permitted to be configured to include a displacement restraint that limits lateral displacement due to the maximum considered earthquake to less than SMI/SM1 times the total design displacement provided that the seismically isolated structure is designed in accordance with the following criteria when more stringent than the requirements of Sec. 13.2:
1. **Maximum considered earthquake** response is calculated in accordance with the dynamic analysis requirements of Sec. 13.4 explicitly considering the nonlinear characteristics of the **isolation system** and the **structure** above the **isolation system**.

2. The ultimate capacity of the **isolation system** and structural elements below the **isolation system** shall exceed the strength and displacement demands of the **maximum considered earthquake**.

3. The **structure** above the **isolation system** is checked for stability and ductility demand of the **maximum considered earthquake**, and

4. The displacement restraint does not become effective at a displacement less than 0.75 times the **total design displacement** unless it is demonstrated by analysis that earlier engagement does not result in unsatisfactory performance.

13.6.2.6 **Vertical-Load Stability:** Each element of the **isolation system** shall be designed to be stable under the maximum vertical load \((1.2D + 1.0L + |E|)\) and the minimum vertical load \((0.8D - |E|)\) at a horizontal displacement equal to the **total maximum displacement**. The **dead load**, \(D\), and the **live load**, \(L\), are specified in Sec. 5.2.7. The seismic load, \(E\), is given by Eq. 5.2.7-1 and 5.2.7-2 where \(S_{DS}\) in these equations is replaced by \(S_{MS}\) and the vertical load due to earthquake, \(Q_E\), shall be based on peak response due to the **maximum considered earthquake**.

13.6.2.7 **Overturning:** The factor of safety against global structural overturning at the **isolation interface** shall not be less than 1.0 for required load combinations. All gravity and seismic loading conditions shall be investigated. **Seismic forces** for overturning calculations shall be based on the **maximum considered earthquake** and \(W\) shall be used for the vertical restoring force.

Local uplift of individual elements is permitted provided the resulting deflections do not cause overstress or instability of the **isolator units** or other **structure** elements.

13.6.2.8 **Inspection and Replacement:**

1. Access for inspection and replacement of all **components** of the **isolation system** shall be provided.

2. A **registered design professional** shall complete a final series of inspections or observations of **structure** separation areas and **components** that cross the **isolation interface** prior to the issuance of the certificate of occupancy for the seismically isolated **structure**. Such inspections and observations shall indicate that the conditions allow free and unhindered displacement of the **structure** to maximum design levels and that all **components** that cross the **isolation interface** as installed are able to accommodate the stipulated displacements.

3. Seismically isolated **structures** shall have a periodic monitoring, inspection and maintenance program for the **isolation system** established by the **registered design professional** responsible for the design of the system.

4. Remodeling, repair or retrofitting at the **isolation system** interface, including that of **components** that cross the **isolation interface**, shall be performed under the direction of a **registered design professional**.
13.6.2.9 **Quality Control:** A quality control testing program for *isolator units* shall be established by the *registered design professional* responsible for the structural design in accordance with Sec. 3.2.1.

13.6.3 **Structural System:**

13.6.3.1 **Horizontal Distribution of Force:** A horizontal diaphragm or other structural elements shall provide continuity above the *isolation interface* and shall have adequate strength and ductility to transmit forces (due to nonuniform ground motion) from one part of the *structure* to another.

13.6.3.2 **Building Separations:** Minimum separations between the isolated *structure* and surrounding retaining *walls* or other fixed obstructions shall not be less than the *total maximum displacement*.

13.6.3.3 **Nonbuilding Structures:** These shall be designed and constructed in accordance with the requirements of Chapter 14 using *design displacements* and forces calculated in accordance with Sec. 13.3 or 13.4.

13.7 **FOUNDATIONS:** Foundations shall be designed and constructed in accordance with the requirements of Chapter 7 using design forces calculated in accordance with Sec. 13.3 or 13.4, as appropriate.

13.8 **DESIGN AND CONSTRUCTION REVIEW:**

13.8.1 **General:** A design review of the *isolation system* and related test programs shall be performed by an independent team of *registered design professionals* in the appropriate disciplines and others experienced in seismic analysis methods and the theory and application of seismic isolation.

13.8.2 **Isolation System:** *Isolation system* design review shall include, but not be limited to, the following:

1. Review of site-specific seismic criteria including the development of site-specific spectra and ground motion time histories and all other design criteria developed specifically for the project;
2. Review of the preliminary design including the determination of the *total design displacement* of the *isolation system design displacement* and the lateral force design level;
3. Overview and observation of prototype testing (Sec. 13.9);
4. Review of the final design of the entire structural system and all supporting analyses; and
5. Review of the *isolation system* quality control testing program (Sec. 13.6.2.9).

13.9 **REQUIRED TESTS OF THE ISOLATION SYSTEM:**

13.9.1 **General:** The deformation characteristics and damping values of the *isolation system* used in the design and analysis of seismically isolated *structures* shall be based on tests of a selected sample of the *components* prior to construction as described in this section.
The *isolation system components* to be tested shall include the wind-restraint system if such a system is used in the design.

The tests specified in this section are for establishing and validating the design properties of the *isolation system* and shall not be considered as satisfying the manufacturing quality control tests of Sec. 13.6.2.9.

13.9.2 Prototype Tests:

13.9.2.1 General: Prototype tests shall be performed separately on two full-size specimens (or sets of specimens, as appropriate) of each predominant type and size of *isolator unit* of the *isolation system*. The test specimens shall include the wind restraint system as well as individual *isolator units* if such systems are used in the design. Specimens tested shall not be used for construction unless accepted by the registered design professional.

13.9.2.2 Record: For each cycle of tests, the force-deflection behavior of the test specimen shall be recorded.

13.9.2.3 Sequence and Cycles: The following sequence of tests shall be performed for the prescribed number of cycles at a vertical load equal to the average *dead load* plus one-half the effects due to *live load* on all *isolator units* of a common type and size:

1. Twenty fully reversed cycles of loading at a lateral force corresponding to the wind design force;
2. Three fully reversed cycles of loading at each of the following increments of the *total design displacement* -- $0.25D_p$, $0.5D_p$, $1.0D_p$, and $1.0D_M$;
3. Three fully reversed cycles of loading at the *total maximum displacement*, $1.0D_{TM}$; and
4. $30S_D B_p / S_D$, but not less than ten, fully reversed cycles of loading at $1$ *total design displacement*, $1.0D_{TD}$.

If an *isolator unit* is also a vertical-load-carrying element, then Item 2 of the sequence of cyclic tests specified above shall be performed for two additional vertical load cases: $1.12D + 0.5L + |E|$ and $2.08D - |E|$ where *dead load*, $D$, and *live load*, $L$, are specified in Sec. 5.2.7. The seismic load, $E$, is given by Eq. 5.2.7-1 and 5.2.7-2 and the load increment due to earthquake overturning, $Q_E$, shall be equal to or greater than the peak earthquake vertical force response corresponding to the test displacement being evaluated. In these tests, the combined vertical load shall be taken as the typical or average downward force on all *isolator units* of a common type and size.

13.9.2.4 Units Dependent on Loading Rates: If the force-deflection properties of the *isolator units* are dependent on the rate of loading, then each set of tests specified in Sec. 13.9.2.3 shall be performed dynamically at a frequency equal to the inverse of the effective period, $T_D$.

If reduced-scale prototype specimens are used to quantify rate-dependent properties of isolators, the reduced-scale prototype specimens shall be of the same type and material and be manufactured with the same processes and quality as full-scale prototypes and shall be tested at a frequency that represents full-scale prototype loading rates.
The force-deflection properties of an *isolator unit* shall be considered to be dependent on the rate of loading if there is greater than a plus or minus 15 percent difference in the effective stiffness and the *effective damping* at the *design displacement* when tested at a frequency equal to the inverse of the effective period, $T_{p}$, of the isolated *structure* and when tested at any frequency in the range of 0.1 to 2.0 times the inverse of the effective period, $T_{p}$, of the isolated *structure*.

**13.9.2.5 Units Dependent on Bilateral Load:** If the force-deflection properties of the *isolator units* are dependent on bilateral load, the tests specified in Sec. 13.9.2.3 and 13.9.2.4 shall be augmented to include bilateral load at the following increments of the *total design displacement*: 0.25 and 1.0, 0.50 and 1.0, 0.75 and 1.0, and 1.0 and 1.0.

*Exception:* If reduced-scale prototype specimens are used to quantify bilateral-load-dependent properties, then such specimens shall be of the same type and material and manufactured with the same processes and quality as full-scale prototypes.

The force-deflection properties of an *isolator unit* shall be considered to be dependent on bilateral load if the bilateral and unilateral force-deflection properties have greater than a plus or minus 15 percent difference in effective stiffness at the *design displacement*.

**13.9.2.6 Maximum and Minimum Vertical Load:** *Isolator units* that carry vertical load shall be statically tested for the maximum and minimum vertical load at the *total maximum displacement*. In these tests, the combined vertical load, $1.2D + 1.0L + |E|$, shall be taken as the maximum vertical force, and the combined vertical load, $0.8D - |E|$, shall be taken as the minimum vertical force, on any one isolator of a common type and size. The *dead load*, $D$, and *live load*, $L$, are specified in Sec. 5.2.7. The seismic load, $E$, is given by Eq. 5.2.7-1 and 5.2.7-2, where $S_{DS}$ in these equations is replaced by $S_{MS}$, and the load increment due to earthquake overturning, $Q_{E}$, shall be equal to or greater than the peak earthquake vertical force response corresponding to the *maximum considered earthquake*.

**13.9.2.7 Sacrificial-Wind-Restraint Systems:** If a sacrificial-wind-restraint system is to be utilized, the ultimate capacity shall be established by test.

**13.9.2.8 Testing Similar Units:** The prototype tests are not required if an *isolator unit* is of similar dimensional characteristics and of the same type and material as a prototype *isolator unit* that has been previously tested using the specified sequence of tests.

**13.9.3 Determination of Force-Deflection Characteristics:** The force-deflection characteristics of the *isolation system* shall be based on the cyclic load tests of isolator prototypes specified in Sec. 13.9.2.

As required, the effective stiffness of an *isolator unit*, $k_{eff}$, shall be calculated for each cycle of loading by the equation:

$$k_{eff} = \frac{|F^+| + |F^-|}{|\Delta^+| + |\Delta^-|} \quad (13.9.3-1)$$
where $F^+$ and $F^-$ are the positive and negative forces at $\Delta^+$ and $\Delta^-$, respectively.

As required, the effective damping, $\beta_{eff}$, of an isolator unit shall be calculated for each cycle of loading by the equation:

$$
\beta_{eff} = \frac{2}{\pi} \left[ \frac{E_{\text{loop}}}{k_{\text{eff}} \left( |\Delta^+| + |\Delta^-| \right)^2} \right]
$$

(13.9.3-2)

where the energy dissipated per cycle of loading, $E_{\text{loop}}$, and the effective stiffness, $k_{\text{eff}}$, shall be based on peak test displacements of $\Delta^+$ and $\Delta^-.$

### 13.9.4 Test Specimen Adequacy:

The performance of the test specimens shall be assessed as adequate if the following conditions are satisfied:

1. The force-deflection plots of all tests specified in Sec. 13.9.2 have a positive incremental force carrying capacity.
   
   1.1. For each increment of test displacement specified in Item 2 of Sec. 13.9.2.3 and for each vertical load case specified in Sec. 13.9.2.3:
      
      There is no greater than a plus or minus 15 percent difference between the effective stiffness at each of the three cycles of test and the average value of effective stiffness for each test specimen;
   
   1.2. For each increment of test displacement specified in Item 2 of Sec. 13.9.2.3 and for each vertical load case specified in Sec. 13.9.2.3:
      
      There is no greater than a 15 percent difference in the average value of effective stiffness of the two test specimens of a common type and size of the isolator unit over the required three cycles of test;

2. For each specimen there is no greater than a plus or minus 20 percent change in the initial effective stiffness of each test specimen over the $30S_{Df}B_{f}/S_{DS}$, but not less than 10, cycles of test specified in Item 3 of Sec. 13.9.2.3;

3. For each specimen there is no greater than a 20 percent decrease in the initial effective damping over for the $30S_{Df}B_{f}/S_{DS}$, but not less than 10, cycles of test specified in Item 3 of Sec. 13.9.2.3; and

4. All specimens of vertical-load-carrying elements of the isolation system remain stable up to the total maximum displacement for static load as prescribed in Sec. 13.9.2.6.

### 13.9.5 Design Properties of the Isolation System:

#### 13.9.5.1 Maximum and Minimum Effective Stiffness:

At the design displacement, the maximum and minimum effective stiffness of the isolated system, $k_{D_{\text{max}}}$ and $k_{D_{\text{min}}}$, shall be based on the cyclic tests of Item 2 of Sec. 13.9.2.3 and calculated by the equations:
At the maximum displacement, the maximum and minimum effective stiffness of the isolation system, $k_{\text{Dmax}}$ and $k_{\text{Dmin}}$, shall be based on the cyclic tests of Item 2 of Sec. 13.9.3 and calculated by the equations:

$$k_{\text{Dmax}} = \frac{\sum |F_d|_{\text{max}} + \sum |F_d|_{\text{max}}}{2D_D}$$  \hspace{1cm} (13.9.5.1-1)

$$k_{\text{Dmin}} = \frac{\sum |F_d|_{\text{min}} + \sum |F_d|_{\text{min}}}{2D_D}$$  \hspace{1cm} (13.9.5.1-2)

At the maximum displacement, the maximum and minimum effective stiffness of the isolation system, $k_{\text{Mmax}}$ and $k_{\text{Mmin}}$, shall be based on the cyclic tests of Item 2 of Sec. 13.9.3 and calculated by the equations:

$$k_{\text{Mmax}} = \frac{\sum |F_m|_{\text{max}} + \sum |F_m|_{\text{max}}}{2D_M}$$  \hspace{1cm} (13.9.5.1-3)

$$k_{\text{Mmin}} = \frac{\sum |F_m|_{\text{min}} + \sum |F_m|_{\text{min}}}{2D_M}$$  \hspace{1cm} (13.9.5.1-4)

The maximum effective stiffness of the isolation system, $k_{\text{Dmax}}$ (or $k_{\text{Mmax}}$), shall be based on forces from the cycle of prototype testing at a test displacement equal to $D_D$ (or $D_M$) that produces the largest value of effective stiffness. Minimum effective stiffness of the isolation system, $k_{\text{Dmin}}$ (or $k_{\text{Mmin}}$), shall be based on forces from the cycle of prototype testing at a test displacement equal to $D_D$ (or $D_M$) that produces the smallest value of effective stiffness.

For isolator units that are found by the tests of Sec. 13.9.3, 13.9.4 and 13.9.5 to have force-deflection characteristics that vary with vertical load, rate of loading or bilateral load, respectively, the values of $k_{\text{Dmax}}$ and $k_{\text{Mmax}}$ shall be increased and the values of $k_{\text{Dmin}}$ and $k_{\text{Mmin}}$ shall be decreased, as necessary, to bound the effects of measured variation in effective stiffness.

13.9.5.2 Effective Damping: At the design displacement, the effective damping of the isolation system, $\beta_d$, shall be based on the cyclic tests of Item 2 of Sec. 13.9.3 and calculated by the equation:
\[ \beta_D = \frac{1}{2 \pi} \left[ \frac{\sum E_D}{k_{D_{\text{max}}} D_D^2} \right] \quad (13.9.5.2-1) \]

In Eq. 13.9.5.2-1, the total energy dissipated per cycle of design displacement response, \( \sum E_D \), shall be taken as the sum of the energy dissipated per cycle in all isolator units measured at a test displacement equal to \( D_D \). The total energy dissipated per cycle of design displacement response, \( \sum E_D \), shall be based on forces and deflections from the cycle of prototype testing at test displacement \( D_D \) that produces the smallest value of effective damping.

At the maximum displacement, the effective damping of the isolation system, \( \beta_M \), shall be based on the cyclic tests of Item 2 of Sec. 13.9.3 and calculated by the equation:

\[ \beta_M = \frac{1}{2 \pi} \left[ \frac{\sum E_M}{k_{M_{\text{max}}} D_M^2} \right] \quad (13.9.5.2-2) \]

In Eq. 13.9.5.2-2, the total energy dissipated per cycle of design displacement response, \( \sum E_M \), shall be taken as the sum of the energy dissipated per cycle in all isolator units measured at a test displacement equal to \( D_M \). The total energy dissipated per cycle of maximum displacement response, \( \sum E_M \), shall be based on forces and deflections from the cycle of prototype testing at test displacement \( D_M \) that produces the smallest value of effective damping.
Appendix to Chapter 13

STRUCTURES WITH DAMPING SYSTEMS

PREFACE: Appendix A13 is an entirely new addition to the 2000 Provisions and contains design criteria, analysis methods, and testing recommendations that have no or only limited history of use.

The appendix is intended for the trial use by design professionals, code groups, and regulatory agencies. Design (peer) review is recommended for all structures with a damping system and should be considered essential when this appendix is used as a basis for regulating or approving the design of actual construction.

13A.1 GENERAL: Every structure with a damping system and every portion thereof shall be designed and constructed in accordance with the requirements of this appendix and the applicable requirements of Chapter 1.

   Exception: Motion and accelerations of seismically isolated structures which contain damping devices across the plane of isolation shall be determined in accordance with the provisions of Chapter 13. Testing and strength requirements of damping devices and other elements of the damping system shall be determined in accordance with the applicable provisions of this Appendix.

13A.2 CRITERIA SELECTION:

13A.2.1 Basis for Design: The procedure and limitations for the design of structures with a damping system shall be determined considering zoning, site characteristics, vertical acceleration, cracked section properties of concrete and masonry members, Seismic Use Group, configuration, structural system, and height in accordance with Sec. 5.2, except as noted below.

13A.2.2 Seismic Use Group: All portions of the structure shall be assigned a Seismic Use Group in accordance with the requirements of Sec. 1.3.

13A.2.3 Seismic Design Category: Each structure shall be assigned to a Seismic Design Category based on the Seismic Use Group and the design spectral response acceleration in accordance with Sec. 4.2.

   Exception: Seismic Design Category A structures with a damping system shall be designed using the design spectral response acceleration determined in accordance with Sec. 4.1.2.5 and the analysis methods and design provisions required for Seismic Design Category B structures.

13A.2.4 Configuration Requirements: Structure design shall consider the combination of forces that occur in the basic seismic-force-resisting system and the damping system, as defined in the following sections.
13A.2.4.1 Seismic-Force-Resisting System: Structures that contain a damping system are required to have a basic seismic-force-resisting system that, in each lateral direction, shall conform to one of the types indicated in Table 5.2.2.

The design of the seismic-force-resisting system in each direction shall comply with the requirements of Sec.13A.7.1 and the following:

1. The materials, detailing, construction and inspection of the seismic-force-resisting system shall meet all applicable requirements defined by the Seismic Design Category.

2. The lateral stiffness of the seismic-force-resisting system used to determine elastic periods and displacements shall include the modeling requirements of Sec. 5.3.7 and 5.4.2.

3. The seismic base shear used for design of the seismic-force-resisting system shall not be less than $V_{min}$, where $V_{min}$ is determined as the greater of the following values:

\[
V_{min} = \frac{V}{B_{V+1}} \quad (13A.2.4.1-1)
\]

\[
V_{min} = 0.75V \quad (13A.2.4.1-2)
\]

where:

- $V$ = total design shear at the base of the structure in the direction of interest, as determined using the procedure of Sec. 5.4, including Sec.5.4.2 (kip or kN), and

- $B_{V+1}$ = numerical coefficient as set forth in Table 13A.3.1 for effective damping equal to the sum of viscous damping in the fundamental mode of vibration of the structure in the direction of interest, $\beta_{v_m}(m = 1)$, plus inherent damping, $\beta_I$, and period of structure equal to $T_1$.

**Exception:** Seismic base shear used for design of the seismic-force-resisting system shall not be taken as less than 1.0V, if either of the following conditions apply:

1. In the direction of interest, the damping system has less than two damping devices on each floor level, configured to resist torsion.

2. The seismic-force-resisting system has a vertical irregularity of Type 1b (Table 5.2.3.3) or a plan irregularity of Type 1b (Table 5.2.3.2).

3. Minimum strength requirements for elements of the seismic-force-resisting-system that are also elements of the damping system or are otherwise required to resist forces from damping devices shall meet the additional requirements of Sec. 13A.7.3.

13A.2.4.2 Damping System: Elements of the damping system shall be designed to remain elastic for design loads including unreduced seismic forces of damping devices as required in Sec. 13A.7.3, unless it is shown by analysis or test that inelastic response of elements would not
adversely affect *damping system* function and inelastic response is limited in accordance with the requirements of Sec. 13A.7.3.4.

13A.2.5 Seismic Criteria:

13A.2.5.1 Design Spectra: Spectra of the *design earthquake* and the *maximum considered earthquake* developed in accordance with Sec. 13.4.4.1 shall be used for the design and analysis of all *structures* with a *damping system*. Site-specific design spectra shall be developed and used for design of *structures* with a *damping system* if any one of the following conditions apply:

1. The *structure* is located on a Class F site or
2. The *structure* is located at a site with $S_f$ greater than 0.60.

13A.2.5.2 Time Histories: Ground-motion time histories of the *design earthquake* and the *maximum considered earthquake* developed in accordance with Sec. 13.4.4.2 shall be used for design and analysis of all *structures* with a *damping system* if either of the following conditions apply:

1. The *structure* is located at a site with $S_f$ greater than 0.60.
2. The *damping system* is explicitly modeled and analyzed using the time history analysis method.

13A.2.6 Selection of Analysis Procedure:

13A.2.6.1 General: A structural analysis shall be made for all *structures* with a *damping system* in accordance with the requirements of this section. The structural analysis shall use linear procedures, nonlinear procedures, or a combination of linear and nonlinear procedures as described below.

The *seismic-force-resisting system* shall be designed using the procedures of either Sec. 13A.2.6.2 or Sec. 13A.2.6.3.

The *damping system* may be designed using the procedures of either Sec. 13A.2.6.2 or 13A.2.6.3, subject to the limitations set forth in these sections. *Damping systems* not meeting these limitations shall be designed using the nonlinear analysis methods as required in Sec. 13A.6.

13A.2.6.2 Equivalent Lateral Force Analysis Procedure: *Structures* with a *damping system* designed using the equivalent lateral force analysis procedure of Sec. 13A.4 shall be subject to the following limitations:

1. In the direction of interest, the *damping system* has at least two *damping devices* in each story, configured to resist torsion.
2. The total effective damping of the fundamental mode, $\beta_{md}(m = 1)$, of the *structure* in the direction of interest is not greater than 35 percent of critical.
3. The *seismic-force-resisting system* does not have a vertical irregularity of Type 1a, 1b, 2, or 3 (Table 5.2.3.3) or a plan irregularity of Type 1a or 1b (Table 5.2.3.2).
4. Floor diaphragms are rigid (Sec. 5.2.31).
5. The height of the structure above the base does not exceed 100 ft (30 m).

6. Peak dynamic response of the structure and elements of the damping system are confirmed by nonlinear time history analysis, when required by Sec. 13A.2.6.4.3.

13A.2.6.3 Response Spectrum Analysis: Structures with a damping system meeting the limitations of Sec. 13A.2.6.2 may be designed using the response spectrum analysis procedure of Sec. 13A.5 and structures not meeting the limitations of Sec. 13A.2.6.2 shall be designed using the response spectrum analysis procedure of Sec. 13A.5, subject to the following limitations:

1. In the direction of interest, the damping system has at least two damping devices in each story, configured to resist torsion,

2. The total effective damping of the fundamental mode, $\beta_m D (m = 1)$, of the structure in the direction of interest is not greater than 35 percent of critical, and

3. Peak dynamic response of the structure and elements of the damping system are confirmed by nonlinear time history analysis, when required by Sec. 13A.2.6.4.3.

13A.2.6.4 Nonlinear Analysis:

13A.2.6.4.1 General: Nonlinear analysis procedures of Sec.13A.6 are permitted for design of all structures with damping systems and shall be used for design of structures with damping systems not meeting linear analysis criteria of Sec. 13A.2.6.3.

Nonlinear time history analysis shall be used to confirm peak dynamic response of the structure and elements of the damping system if the structure is located at a site with $S_1$ greater than 0.60g.

The nonlinear force-deflection characteristics of elements of the seismic-force-resisting system shall be modeled as required by Sec. 5.7.1 and 5.8.1. The nonlinear force-deflection characteristics of damping devices shall be modeled, as required, to explicitly account for device dependence on frequency, amplitude and duration of seismic loading.

13A.2.6.4.2 Nonlinear Static Analysis: Structures with a damping system designed using the nonlinear static analysis procedure of Sec. 13A.6 shall be subject to the following limitations:

1. Peak dynamic response of the structure and elements of the damping system is confirmed by nonlinear time history analysis, when required by Sec. 13A.2.6.4.3.

13A.2.6.4.3 Nonlinear Time History Analysis: Structures with a damping system may be designed using the nonlinear time history analysis procedure of Sec. 13A.6 without limitation.

Nonlinear time history analysis shall be used to confirm peak dynamic response of the structure and elements of the damping system for structures with a damping system if the following conditions applies:

1. The structure is located at site with $S_1$ greater than 0.60.

13A.3 DAMPED RESPONSE MODIFICATION:

13A.3.1 General: As required in Sec. 13A.4 and 13A.5, response of the structure shall be modified for the effects of the damping system using coefficients prescribed in Table 13A.3.1.
Table 13A.3.1 Damping Coefficient, $B_{V+I}$, $B_{ID}$, $B_{RD}$, $B_{IM}$, $B_{mD}$ or $B_{mM}$

<table>
<thead>
<tr>
<th>Effective Damping, $\beta$</th>
<th>Period of the Structure $\geq T_s/5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\leq 2%$</td>
<td>0.8</td>
</tr>
<tr>
<td>$5%$</td>
<td>1.0</td>
</tr>
<tr>
<td>$10%$</td>
<td>1.2</td>
</tr>
<tr>
<td>$20%$</td>
<td>1.5</td>
</tr>
<tr>
<td>$30%$</td>
<td>1.8</td>
</tr>
<tr>
<td>$40%$</td>
<td>2.1</td>
</tr>
<tr>
<td>$50%$</td>
<td>2.4</td>
</tr>
<tr>
<td>$60%$</td>
<td>2.7</td>
</tr>
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<td>$70%$</td>
<td>3.0</td>
</tr>
<tr>
<td>$80%$</td>
<td>3.3</td>
</tr>
<tr>
<td>$90%$</td>
<td>3.6</td>
</tr>
<tr>
<td>$\geq 100%$</td>
<td>4.0</td>
</tr>
</tbody>
</table>

1The damping coefficient is equal to 1.0 at a period of the structure equal to 0 second for all values of effective damping. Interpolation may be used for intermediate values of effective damping at periods of the structure between 0 second and $T_s/5$ seconds.

13A.3.2 Effective Damping: The effective damping at the design displacement, $\beta_{mD}$, and at the maximum displacement, $\beta_{mM}$, of the mth mode of vibration of the structure in the direction under consideration shall be calculated as follows:

$$\beta_{mD} = \beta_l + \beta_{vm} \sqrt{\mu_D} + \beta_{HD}$$  \hspace{1cm} (13A.3.2-1)

$$\beta_{mM} = \beta_l + \beta_{vm} \sqrt{\mu_M} + \beta_{HM}$$  \hspace{1cm} (13A.3.2-2)

where:

$\beta_{HD} =$ component of effective damping of the structure in the direction of interest due to post-yield hysteretic behavior of the seismic-force-resisting system and elements of the damping system at effective ductility demand, $\mu_D$;

$\beta_{HM} =$ component of effective damping of the structure in the direction of interest due to post-yield hysteretic behavior of the seismic-force-resisting system and elements of the damping system at effective ductility demand, $\mu_M$;

$\beta_l =$ component of effective damping of the structure due to the inherent dissipation of energy by elements of the structure, at or just below the effective yield displacement of the seismic-force-resisting system;
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\[ \beta_{Vm} = \text{component of effective damping of the } m^{th} \text{ mode of vibration of the } structure \text{ in the direction of interest due to viscous dissipation of energy by the damping system, at or just below the effective yield displacement of the seismic-force-resisting system;} \]

\[ \mu_D = \text{effective ductility demand on the seismic-force-resisting system in the direction of interest due to the design earthquake;} \text{ and} \]

\[ \mu_M = \text{effective ductility demand on the seismic-force-resisting system in the direction of interest due to the maximum considered earthquake.} \]

Unless analysis or test data supports other values, the effective ductility demand of higher modes of vibration in the direction of interest shall be taken as 1.0.

13A.3.2.1 Inherent Damping: Inherent damping, \( \beta_p \), shall be based on the material type, configuration and behavior of the structure and nonstructural components responding dynamically at or just below yield of the seismic-force-resisting system. Unless analysis or test data supports other values, inherent damping shall be taken as not greater than 5 percent of critical for all modes of vibration.

13A.3.2.2 Hysteretic Damping: Hysteretic damping of the seismic-force-resisting system and elements of the damping system shall be based either on test or analysis, or in accordance with the following equations:

\[ \beta_{HD} = q_H (0.64 - \beta_I) \left(1 - \frac{1}{\mu_D}\right) \quad (13A.3.2.2-1) \]

\[ \beta_{HM} = q_H (0.64 - \beta_I) \left(1 - \frac{1}{\mu_M}\right) \quad (13A.3.2.2-2) \]

where:

\[ q_H = \text{hysteresis loop adjustment factor, as defined in Sec. 13A.3.3;} \]

\[ \mu_D = \text{effective ductility demand on the seismic-force-resisting system in the direction of interest due to the design earthquake, as defined in Sec. A.13.3.4;} \text{ and} \]

\[ \mu_M = \text{effective ductility demand on the seismic-force-resisting system in the direction of interest due to the maximum considered earthquake, as defined in Sec. 13A.3.4.} \]

Unless analysis or test data supports other values, the hysteretic damping of higher modes of vibration in the direction of interest shall be taken as zero.

13A.3.2.3 Viscous Damping: Viscous damping of the \( m^{th} \) mode of vibration of the structure, \( \beta_{Vm} \), shall be calculated as follows:
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\[ \beta_{im} = \frac{\sum W_{mj}}{4\pi W_m} \]  

(13A.3.2.3-1)

\[ W_m = \frac{1}{2} \sum F_{im} \delta_{im} \]  

(13A.3.2.3-2)

where:

- \( W_{mj} \) = work done by \( j \)th damping device in one complete cycle of dynamic response corresponding to the \( m \)th mode of vibration of the structure in the direction of interest at modal displacements, \( \delta_{im} \);
- \( W_m \) = maximum strain energy in the \( m \)th mode of vibration of the structure in the direction of interest at modal displacements, \( \delta_{im} \);
- \( F_{im} \) = \( m \)th mode inertial force at Level \( i \) (or mass point); and
- \( \delta_{im} \) = deflection of Level \( i \) (or mass point) in the \( m \)th mode of vibration at the center of rigidity of the structure in the direction under consideration.

Viscous modal damping of displacement-dependent damping devices shall be based on a response amplitude equal to the effective yield displacement of the structure.

The calculation of the work done by individual damping devices shall consider orientation and participation of each device with respect to the mode of vibration of interest. The work done by individual damping devices shall be reduced as required to account for the flexibility of elements, including pins, bolts, gusset plates, brace extensions, and other components that connect damping devices to other elements of the structure.

13A.3.3 Hysteresis Loop Adjustment Factor: Hysteretic damping of the seismic-force-resisting system and elements of the damping system shall consider pinching and other effects that reduce the area of the hysteresis loop during repeated cycles of earthquake demand. Unless analysis or test data support other values, the fraction of full hysteretic loop area of the seismic-force-resisting system used for design shall be taken as equal to the factor, \( q_H \), as defined below:

\[ q_H = 0.67 \frac{T_S}{T_i} \]  

(13A.3.3-1)

where:

- \( T_s \) = period, in seconds, defined by the ratio, \( S_{Dj}/S_{DS} \), and
- \( T_i \) = period, in seconds of the fundamental mode of vibration of the structure in the direction of the interest.

The value of \( q_H \) shall not be taken as greater than 1.0, and need not be taken as less than 0.5.
13A.3.4 Effective Ductility Demand: The effective ductility demand of seismic-force-resisting system due to the design earthquake, $\mu_D$, and due to the maximum considered earthquake, $\mu_M$, shall be calculated as the ratio of the fundamental mode displacement, $D_{1D}$ or $D_{1M}$, to effective yield displacement, $D_Y$:

\[
\mu_D = \frac{D_{1D}}{D_Y} \geq 1.0 \quad (13A.3.4-1)
\]

\[
\mu_M = \frac{D_{1M}}{D_Y} \geq 1.0 \quad (13A.3.4-2)
\]

\[
D_Y = \left(\frac{g}{4\pi^2}\right) \left(\frac{\Omega_s C_d}{R}\right) \Gamma_1 C_{sl} T_1^2
\]

(13A.3.4-3)

where:

- $D_{1D}$ = fundamental mode design displacement at the center of rigidity of the roof level of the structure in the direction under consideration, Sec. 13A.4.4.3 (in. or mm),
- $D_{1M}$ = fundamental mode maximum displacement at the center of rigidity of the roof level of structure in the direction under consideration, Sec. 13A.4.4.6 (in. or mm),
- $D_Y$ = displacement at the center of rigidity of the roof level of the structure at the effective yield point of the seismic-force-resisting system, Sec. 13A.3.4 (in. or mm),
- $R$ = response modification factor from Table 5.2.2,
- $C_d$ = deflection amplification factor from Table 5.2.2,
- $\Omega_s$ = system overstrength factor from Table 5.2.2,
- $\Gamma_1$ = participation factor of the fundamental mode of vibration of the structure in the direction of interest, Sec. 13A.4.3.3 or Sec. 13A.5.3.3 ($m=1$),
- $C_{sl}$ = seismic response coefficient (dimensionless) of the fundamental mode of vibration of the structure in the direction of interest, Sec. 13A.4.3.4 or Sec. 13A.5.3.4 ($m=1$), and
- $T_1$ = period, in seconds, of the fundamental mode of vibration of the structure in the direction of interest.

Design earthquake ductility demand, $\mu_D$, shall not exceed the maximum value of effective ductility demand, $\mu_{max}$, given in Sec. 13A.3.5.
13A.3.5 Maximum Effective Ductility Demand: For determination of the hysteresis loop adjustment factor, hysteretic damping and other parameters, ductility demand used for design of the structure shall not exceed the maximum value of effective ductility demand, $\mu_{\text{max}}$, as defined below:

For $T_{ID} < T_S$:

$$
\mu_{\text{max}} = \frac{1}{2} \left( \frac{R}{\Omega_o I} \right)^2 + 1
$$

(13A.3.5-1)

For $T_I \geq T_S$:

$$
\mu_{\text{max}} = \frac{R}{\Omega_o I}
$$

(13A.3.5-2)

where:

$I$ = the occupancy importance factor determined in accordance with Sec. 1.4.

$T_{ID}$ = effective period, in seconds, of the fundamental mode of vibration of the structure at the design displacement in the direction under consideration.

For periods: $T_I \leq T_S \leq T_{ID}$, interpolation shall be used to determine $\mu_{\text{max}}$.

13A.4 EQUIVALENT LATERAL FORCE ANALYSIS PROCEDURE:

13A.4.1 General: This section provides required minimum standards for equivalent lateral force analysis of structures with a damping system. For purposes of analysis, the structure is considered to be fixed at the base. See Sec. 13A.2.6 for limitations on the use of this procedure.

Seismic base shear and lateral forces at floors used for design of the seismic-force-resisting system shall be based on the procedures of Sec. 13A.4.3. Seismic forces, displacements and velocities used for design of the damping system shall be based on the procedures of Sec. 13A.4.4.

The load combinations and acceptance criteria of Sec. 13A.7 shall be used to check design responses of seismic-force-resisting and damping systems, respectively.

13A.4.2 Modeling Requirements: Elements of the seismic-force-resisting system shall be modeled in a manner consistent with the requirements of Sec. 5.4.

Elements of the damping system shall be modeled as required to determine design forces transferred from damping devices to both the ground and the seismic-force-resisting system. The effective stiffness of velocity-dependent damping devices shall be modeled.

Damping devices need not be explicitly modeled provided effective damping is calculated in accordance with the procedures of Sec. 13A.3 and used to modify response as required in Sec. 13A.4.3 and 13A.4.4.

The stiffness and damping properties of the damping devices used in the models shall be based on or verified by testing of the damping devices as specified in Sec. 13A.10.
13A.4.3 Seismic-Force-Resisting-System Design Response:

13A.4.3.1 Seismic Base Shear: The seismic base shear, $V$, of the seismic-force-resisting system in a given direction shall be determined as the combination of the two modal components, $V_1$ and $V_R$, in accordance with the following equation:

$$V = \sqrt{V_1^2 + V_R^2} \geq V_{\text{min}}$$  \hspace{1cm} (13A.4.3.1-1)

where:

- $V_1$ = design value of the seismic base shear of the fundamental mode in a given direction of response (kip or kN),
- $V_R$ = design value of the seismic base shear of the residual mode in a given direction (kip or kN), and
- $V_{\text{min}}$ = minimum allowable value of base shear permitted for design of the seismic-force-resisting-system of the structure in direction of the interest (kip or kN).

13A.4.3.2 Fundamental Mode Base Shear: Fundamental mode base shear, $V_1$, shall be determined in accordance with the following equation:

$$V_1 = C_{S1} \overline{W}_1$$ \hspace{1cm} (13A.4.3.2-1)

where:

- $\overline{W}_1$ = the effective fundamental mode gravity load including portions of the live load as defined by Eq. 5.5.5-2 for $m = 1$ (kip or kN).

13A.4.3.3 Fundamental Mode Properties: Fundamental mode shape, $\phi_{ij}$, and participation factor, $\Gamma_1$, shall be determined by either dynamic analysis of elastic structural properties and deformational characteristics of the resisting elements or in accordance with the following equations:

$$\phi_{ij} = \frac{h_i}{h_r}$$ \hspace{1cm} (13A.4.3.3-1)

$$\Gamma_1 = \frac{\overline{W}_1}{\sum_{i=1}^{n} w_i \phi_{ij}}$$ \hspace{1cm} (13A.4.3.3-2)

where:

- $h_i$ = the height of the structure above the base to Level $i$ (ft or m),
\[ h_r = \text{the height of the structure above the base to the roof level (ft or m), and} \]
\[ w_i = \text{the portion of the total gravity load, } W, \text{ located or assigned to Level } i. \]

The fundamental period, \( T_1 \), shall be determined either by dynamic analysis of elastic structural properties and deformational characteristics of the resisting elements, or in accordance with the following equation:

\[
T_1 = 2\pi \sqrt{\frac{\sum_{i=1}^{n} w_i \delta_i^2}{g \sum_{i=1}^{n} f_i \delta_i}}
\]  

(13A.4.3.3-3)

where:
\( f_i = \text{lateral force at Level } i \text{ of the structure distributed in accordance with Eq. 5.4.3-2, and} \)
\( \delta_i = \text{elastic deflection at Level } i \text{ of the structure due to applied lateral forces } f_i. \)

13A.4.3.4 Fundamental Mode Seismic Response Coefficient: The fundamental mode seismic response coefficient, \( C_{s1} \), shall be determined in accordance with the following equations:

For \( T_{1D} < T_S \):

\[
C_{s1} = \left( \frac{R}{C_d} \right) \frac{S_{DS}}{\Omega_a B_{1D}}
\]  

(13A.4.3.4-1)

For \( T_{1D} \geq T_S \):

\[
C_{s1} = \left( \frac{R}{C_d} \right) \frac{S_{D1}}{T_{1D} \left( \Omega_a B_{1D} \right)}
\]  

(13A.4.3.4-2)

where:
\( S_{DS} = \text{the design spectral response acceleration in the short period range as determined from Sec. 4.1.2.5,} \)
\( S_{D1} = \text{the design spectral response acceleration at a period of 1 second as determined from Sec. 4.1.2.5,} \)
\( B_{1D} = \text{numerical coefficient as set forth in Table 13A.3.1 for effective damping equal to } \beta_{mD} (m = 1) \text{ and period of the structure equal to } T_{1D}. \)
13A.4.3.5 Effective Fundamental Mode Period Determination: The effective fundamental mode period at the design earthquake, $T_{ID}$, and at the maximum considered earthquake, $T_{IM}$, shall be based either on explicit consideration of the post-yield force deflection characteristics of the structure or in accordance with the following equations:

$$T_{ID} = T \sqrt[\mu]{D}$$  \hspace{1cm} (13A.4.3.5-1)

$$T_{IM} = T \sqrt[\mu]{M}$$  \hspace{1cm} (13A.4.3.5-2)

where:

$T_{IM} = \text{effective period, in seconds, of the fundamental mode of vibration of the structure at the maximum displacement in the direction under consideration.}$

13A.4.3.6 Residual Mode Base Shear: Residual mode base shear, $V_R$, shall be determined in accordance with the following equation:

$$V_R = C_{SR} W_R$$  \hspace{1cm} (13A.4.3.6-1)

where:

$C_{SR} = \text{the residual mode seismic response coefficient as determined in Sec. 13A.4.3.8, and}$

$W_R = \text{the effective residual mode gravity load of the structure determined in accordance with Eq. 13A.4.3.7-3 (kip or kN).}$

13A.4.3.7 Residual Mode Properties: Residual mode shape, $\phi_R$, participation factor, $\Gamma_R$, effective gravity load of the structure, $W_R$, and effective period, $T_R$, shall be determined in accordance with the following equations:

$$\phi_R = \frac{1 - \Gamma_R \phi_i}{1 - \Gamma_i}$$  \hspace{1cm} (13A.4.3.7-1)

$$\Gamma_R = 1 - \Gamma_i$$  \hspace{1cm} (13A.4.3.7-2)

$$W_R = W - W_i$$  \hspace{1cm} (13A.4.3.7-3)

$$T_R = 0.4 T_i$$  \hspace{1cm} (13A.4.3.7-4)

13A.4.3.8 Residual Mode Seismic Response Coefficient: The residual mode seismic response coefficient, $C_{SR}$, shall be determined in accordance with the following equation:
\[ C_{SR} = \left( \frac{R}{C_d} \right) \frac{S_{DS}}{\Omega_s B_R} \]  

(A13.4.3.8-1)

where:

\( B_R \quad = \quad \text{Numerical coefficient as set forth in Table 13A.3.1 for effective damping equal to } \beta_r, \text{ and period of the structure equal to } T_R \).

13A.4.3.9 Design Lateral Force: Design lateral force in elements of the seismic-force-resisting system at Level \( i \) due to fundamental mode response, \( F_{i1} \), and residual mode response, \( F_{iR} \), of the structure in the direction of interest shall be determined in accordance with the following equations:

\[ F_{i1} = w_i \phi_{i1} \frac{\Gamma_i}{W_i} V_1 \]  

(13A.4.3.9-1)

\[ F_{iR} = w_i \phi_{iR} \frac{\Gamma_R}{W_R} V_R \]  

(13A.4.3.9-2)

Design forces in elements of the seismic-force-resisting system shall be determined as the square-root-sum-of-squares of the forces due to fundamental and residual modes.

13A.4.4 Damping System Design Response:

13A.4.4.1 General: Design forces in damping devices and other elements of the damping system shall be determined on the basis of the floor deflection, story drift and story velocity response parameters described in the following sections.

Displacements and velocities used to determine maximum forces in damping devices at each story shall account for the angle of orientation from horizontal and consider the effects of increased response due to torsion required for design of the seismic-force-resisting system.

Floor deflections at Level \( i \), \( \delta_{iD} \) and \( \delta_{iR} \), design story drifts, \( \Delta_D \) and \( \Delta_M \), and design story velocities, \( \nabla_D \) and \( \nabla_M \), shall be calculated for both the design earthquake and the maximum considered earthquake, respectively, in accordance with the following sections.

13A.4.4.2 Design Earthquake Floor Deflection: Fundamental and residual mode deflections due to the design earthquake, \( \delta_{i1D} \) and \( \delta_{iRD} \) (in. or mm), at the center of rigidity of Level \( i \) of the structure in the direction of interest shall be determined in accordance with the following equations:

\[ \delta_{i1D} = D_{1D} \phi_{i1} \]  

(13A.4.4.2-1)

\[ \delta_{iRD} = D_{RD} \phi_{iR} \]  

(13A.4.4.2-2)
where:

\[ D_{RD} = \text{Residual mode design displacement at the center of rigidity of the roof level of the structure in the direction under consideration, Sec. 13A.4.4.3 (in. or mm).} \]

The total design earthquake deflection at each floor of the structure in the direction of interest shall be calculated as the square-root-sum-of-squares of fundamental and residual mode floor deflections.

13A.4.4.3 Design Earthquake Roof Displacement: Fundamental and residual mode displacements due to the design earthquake, \( D_{ID} \) and \( D_{IR} \) (in. or mm) at the center of rigidity of the roof level of the structure in the direction of interest shall be determined in accordance with the following equations:

\[
D_{ID} = \left( \frac{g}{4\pi^2} \right) \Gamma_i \left( \frac{S_{DI} T_{ID}}{B_{ID}} \right) \leq \left( \frac{g}{4\pi^2} \right) \Gamma_i \left( \frac{S_{DS} T_{ID}^2}{B_{ID}} \right) \quad (13A.4.4.3-1)
\]

\[
D_{RD} = \left( \frac{g}{4\pi^2} \right) \Gamma_R \left( \frac{S_{DI} T_{IR}}{B_R} \right) \leq \left( \frac{g}{4\pi^2} \right) \Gamma_R \left( \frac{S_{DS} T_{IR}^2}{B_R} \right) \quad (13A.4.4.3-2)
\]

13A.4.4.4 Design Earthquake Story Drift: Design earthquake story drift, \( \Delta_D \), of the structure in the direction of interest shall be calculated in accordance with the following equation:

\[
\Delta_D = \sqrt{\Delta_{ID}^2 + \Delta_{RD}^2} \quad (13A.4.4.4-1)
\]

where:

\[ \Delta_{ID} = \text{design earthquake story drift due to the fundamental mode of vibration of the structure in the direction of interest (in. or mm) and} \]

\[ \Delta_{RD} = \text{design earthquake story drift due to the residual mode of vibration of the structure in the direction of interest (in. or mm).} \]

Modal design earthquake story drifts, \( \Delta_{ID} \) and \( \Delta_{IR} \), shall be determined in accordance with Sec. 5.3.7.1 using the floor deflections of Sec. 13A.4.4.2.

13A.4.4.5 Design Earthquake Story Velocity: Design earthquake story velocity, \( V_D \), of the structure in the direction of interest shall be calculated in accordance with the following equations:

\[
V_D = \sqrt{V_{ID}^2 + V_{RD}^2} \quad (13A.4.4.5-1)
\]

\[
V_{ID} = 2\pi \frac{\Delta_{ID}}{T_{ID}} \quad (13A.4.4.5-2)
\]
\[ \nabla_{RD} = 2\pi \frac{\Delta_{RD}}{T_R} \]  

(13A.4.4.5-3)

where:

\[ \Delta_{1D} = \text{design earthquake story velocity due to the fundamental mode of vibration of the structure in the direction of interest (in./sec or mm/sec), and} \]

\[ \nabla_{RD} = \text{design earthquake story velocity due to the residual mode of vibration of the structure in the direction of interest (in./sec or mm/sec).} \]

**13A.4.4.6 Maximum Earthquake Response:** Total and modal maximum earthquake floor deflections at Level \( i \), design story drift values and design story velocity values shall be based on the formulas of Sec. 13A.4.2, 13A.4.4 and 13A.4.5, respectively, except design earthquake roof displacements shall be replaced by maximum earthquake roof displacements. Maximum earthquake roof displacements shall be calculated in accordance with the following equations:

\[ D_{1M} = \left( \frac{g}{4\pi^2} \right) \Gamma_1 S_{M1} T_{1M} \leq \left( \frac{g}{4\pi^2} \right) \Gamma_1 S_{MS} T_{1M} \]  

(13A.4.4.6-1)

\[ D_{RM} = \left( \frac{g}{4\pi^2} \right) \Gamma_R S_{M1} T_R \leq \left( \frac{g}{4\pi^2} \right) \Gamma_R S_{MS} T_R \]  

(13A.4.4.6-2)

where:

\[ S_{M1} = \text{the maximum considered earthquake, 5 percent damped, spectral response acceleration at a period of 1 second adjusted for site class effects as defined in Sec.4.1.2.} \]

\[ S_{MS} = \text{the maximum considered earthquake, 5 percent damped, spectral response acceleration at short periods adjusted for site class effects as defined in Sec. 4.1.2.} \]

\[ B_{1M} = \text{Numerical coefficient as set forth in Table 13A.3.1 for effective damping equal to } \beta_{mM} (m = 1) \text{ and period of structure equal to } T_{1M}. \]

**13A.5 RESPONSE SPECTRUM ANALYSIS PROCEDURE**

**13A.5.1 General:** This section provides required standards for response spectrum analysis of structures with a damping system. See Sec. 13A.2.6 for limitations on the use of this procedure.

Seismic base shear and lateral forces at floors used for design of the seismic-force-resisting system shall be based on the procedures of Sec. 13A.3.2. Seismic forces, displacements and velocities used for design of the damping system shall be based on the procedures of Sec. 13A.3.

The load combinations and acceptance criteria of Sec. 13A.7 shall be used to check design responses of seismic-force-resisting system and the damping system.
13A.5.2 Modeling and Analysis Requirements:

13A.5.2.1 General: A mathematical model of the seismic-force-resisting system and damping system shall be constructed that represents the spatial distribution of mass, stiffness and damping throughout the structure. The model and analysis shall conform to the requirements of Sec. 5.5.1, 5.5.2, and 5.5.3 for the seismic-force-resisting system and to the requirements of Sec. 13A.5.2.2 for the damping system. The stiffness and damping properties of the damping devices used in the models shall be based on or verified by testing of the damping devices as specified in Sec. 13A.10.

13A.5.2.2 Damping System: The elastic stiffness of elements of the damping system other than damping devices shall be explicitly modeled. Stiffness of damping devices shall be modeled depending on damping device type:

1. Displacement-Dependent Damping Devices: Displacement-dependent damping devices shall be modeled with an effective stiffness that represents damping device force at the response displacement of interest (e.g., design story drift). Alternatively, the stiffness of hysteretic and friction damping devices may be excluded from response spectrum analysis provided design forces in displacement-dependent damping devices, \( Q_{DSD} \), are applied to the model as external loads (Sec. 13A.7.3.2).

2. Velocity-Dependent Damping Devices: Velocity-dependent damping devices that have a stiffness component (e.g., visco-elastic damping devices) shall be modeled with an effective stiffness corresponding to the amplitude and frequency of interest.

13A.5.3 Seismic-Force-Resisting-System Design Response:

13A.5.3.1 Seismic Base Shear: The seismic base shear, \( V \), of the structure in a given direction shall be determined as the combination of modal components, \( V_m \), subject to the limits of the following equation:

\[
V \geq V_{min}
\]  
(13A.5.3.1-1)

The seismic base shear, \( V \), of the structure shall be determined by the square root sum of the squares or complete quadratic combination of modal base shear components, \( V_m \).

13A.5.3.2 Modal Base Shear: Modal base shear of the \( m \)th mode of vibration, \( V_m \), of the structure in the direction of interest shall be determined in accordance with the following equation:

\[
V_m = C_{Sm} W_m
\]  
(13A.5.3.2-1)

where:

\[
C_{Sm} = \text{seismic response coefficient} \text{ (dimensionless)} \text{ of the } m \text{th mode of vibration of the structure in the direction of interest, Sec. 13A.5.3.4 } (m = 1) \text{ or Sec. 13A.5.3.6 } (m > 1) \text{ and}
\]
\[ \bar{W}_m = \text{the effective gravity load of the } m^{th} \text{ mode of vibration of the structure determined in accordance with Eq. 5.4.4-2 (kip or kN).} \]

**13A.5.3.3 Modal Participation Factor:** The modal participation factor of the \( m^{th} \) mode of vibration, \( \Gamma_m \), of the structure in the direction of interest shall be determined in accordance with the following equation:

\[
\Gamma_m = \frac{\bar{W}_m}{\sum_{i=1}^{n} w_i \phi_{im}} \quad (13A.5.3.3-1)
\]

where:

\[ \phi_{im} = \text{displacement amplitude at the } i^{th} \text{ level of the structure for the fixed base condition in the } m^{th} \text{ mode of vibration in the direction of interest, normalized to unity at the roof level.} \]

**13A.5.3.4 Fundamental Mode Seismic Response Coefficient:** The fundamental mode \( (m = 1) \) seismic response coefficient, \( C_{S1} \), in the direction of interest shall be determined in accordance with the following equations:

For \( T_{1D} < T_S \):

\[
C_{S1} = \left( \frac{R}{C_d} \right) \frac{S_{DS}}{\Omega_o B_{1D}} \quad (13A.5.3.4-1)
\]

For \( T_{1D} \geq T_S \):

\[
C_{S1} = \left( \frac{R}{C_d} \right) \frac{S_{D1}}{T_{1D} \left( \Omega_o B_{1D} \right)} \quad (13A.5.3.4-2)
\]

**13A.5.3.5 Effective Fundamental Mode Period Determination:** The effective fundamental mode \( (m = 1) \) period at the design earthquake, \( T_{1D} \), and at the maximum considered earthquake, \( T_{1M} \), shall be based either on explicit consideration of the post-yield nonlinear force deflection characteristics of the structure or determined in accordance with the following equations:

\[
T_{1D} = T_1 \sqrt{\mu_D} \quad (13A.4.3.5-1)
\]

\[
T_{1M} = T_1 \sqrt{\mu_M} \quad (13A.4.3.5-2)
\]

**13A.5.3.6 Higher Mode Seismic Response Coefficient:** Higher mode \( (m > 1) \) seismic response coefficient, \( C_{Sm} \), of the \( m^{th} \) mode of vibration \( (m > 1) \) of the structure in the direction of interest shall be determined in accordance with the following equations:
For $T_m < T_S$ : 

$$C_{Sm} = \left( \frac{R}{C_d} \right) \frac{S_{DS}}{\Omega_m B_{mD}}$$  \hspace{1cm} (13A.5.3.6-1)

For $T_m \geq T_S$ : 

$$C_{Sm} = \left( \frac{R}{C_d} \right) \frac{S_{DL}}{T_m (\Omega_m B_{mD})}$$  \hspace{1cm} (13A.5.3.6-2)

where:

$T_m$ = period, in seconds, of the $m^{th}$ mode of vibration of the structure in the direction under consideration and

$B_{mD}$ = numerical coefficient as set forth in Table 13A.3.1 for effective damping equal to $\beta_{mD}$ and period of the structure equal to $T_m$.

13A.5.3.7 Design Lateral Force: Design lateral force at Level $i$ due to $m^{th}$ mode of vibration, $F_{im}$, of the structure in the direction of interest shall be determined in accordance with the following equation:

$$F_{im} = w_i \phi_{im} \frac{\Gamma_m}{W_m} V_m$$  \hspace{1cm} (13A.5.3.7-1)

Design forces in elements of the seismic-force-resisting system shall be determined by the square root sum of squares or complete quadratic combination of modal design forces.

13A.5.4 Damping System Design Response:

13A.5.4.1 General: Design forces in damping devices and other elements of the damping system shall be determined on the basis of the floor deflection, story drift and story velocity response parameters described in the following sections.

Displacements and velocities used to determine maximum forces in damping devices at each story shall account for the angle of orientation from horizontal and consider the effects of increased response due to torsion required for design of the seismic-force-resisting system.

Floor deflections at Level $i$, $\delta_{id}$ and $\delta_{im}$, design story drifts, $\Delta_d$ and $\Delta_m$, and design story velocities, $V_d$ and $V_m$, shall be calculated for both the design earthquake and the maximum considered earthquake, respectively, in accordance with the following sections.

13A.5.4.2 Design Earthquake Floor Deflection: The deflection of structure due to the design earthquake at Level $i$ in the $m^{th}$ mode of vibration, $\delta_{imD}$ (in. or mm), of the structure in the direction of interest shall be determined in accordance with the following equation:

$$\delta_{imD} = D_{mD} \phi_{im}$$  \hspace{1cm} (13A.5.4.2-1)
The total design earthquake deflection at each floor of the structure shall be calculated by the square root sum of squares or complete quadratic combination of modal design earthquake deflections.

13A.5.4.3 Design Earthquake Roof Displacement: Fundamental \((m = 1)\) and higher mode \((m > 1)\) roof displacements due to the design earthquake, \(D_{1D}\) and \(D_{mD}\) (in. or mm), of the structure in the direction of interest shall be determined in accordance with the following equations:

For \(m = 1\):
\[
D_{1D} = \left( \frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{DI1D} T_{1D}}{B_{1D}} \leq \left( \frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{DS} T_{1D}^2}{B_{1D}}
\]

For \(m > 1\):
\[
D_{mD} = \left( \frac{g}{4\pi^2} \right) \Gamma_m \frac{S_{DmD} T_m}{B_{mD}} \leq \left( \frac{g}{4\pi^2} \right) \Gamma_m \frac{S_{DS} T_m^2}{B_{mD}}
\]

13A.5.4.4 Design Earthquake Story Drift: Design earthquake story drift of the fundamental mode, \(\Delta_{1D}\), and higher modes, \(\Delta_{mD}\) \((m > 1)\), of the structure in the direction of interest shall be calculated in accordance with Sec. 5.3.7.1 using modal roof displacements of Sec. 13A.5.4.3.

Total design earthquake story drift, \(\Delta_D\) (in. or mm), shall be determined by the square root of the sum of squares or complete quadratic combination of modal design earthquake drifts.

13A.5.4.5 Design Earthquake Story Velocity: Design earthquake story velocity of the fundamental mode, \(\nabla_{1D}\), and higher modes, \(\nabla_{mD}\) \((m > 1)\), of the structure in the direction of interest shall be calculated in accordance with the following equations:

For \(m = 1\):
\[
\nabla_{1D} = 2\pi \frac{\Delta_{1D}}{T_{1D}}
\]

For \(m > 1\):
\[
\nabla_{mD} = 2\pi \frac{\Delta_{mD}}{T_m}
\]

Total design earthquake story velocity, \(\nabla_D\) (in/sec or mm/sec), shall be determined by the square root of the sum of squares or complete quadratic combination of modal design earthquake velocities.

13A.5.4.6 Maximum Earthquake Response: Total modal floor deflection at Level \(i\), design story drift values and design story velocity values shall be based on the formulas of Sec. 13A.5.4.2, 13A.5.4.4 and 13A.5.4.5, respectively, except design earthquake roof displacement shall be replaced by maximum earthquake roof displacement. Maximum earthquake roof displacement of the structure in the direction of interest shall be calculated in accordance with the following equations:
For $m = 1$: \[ D_{1M} = \left( \frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{S1T_1}}{B_{1M}} \leq \left( \frac{g}{4\pi^2} \right) \Gamma_1 \frac{S_{S1T_1}}{B_{1M}} \] (13A.5.6.1-1)

For $m > 1$: \[ D_{mM} = \left( \frac{g}{4\pi^2} \right) \Gamma_m \frac{S_{S1T_m}}{B_{mM}} \leq \left( \frac{g}{4\pi^2} \right) \Gamma_m \frac{S_{S1T_m}}{B_{mM}} \] (13A.5.6.1-2)

where: \[ B_{mM} \] = numerical coefficient as set forth in Table 13A.3.1 for effective damping equal to $\beta_{mM}$ and period of the structure equal to $T_m$.

### 13A.6 NONLINEAR ANALYSIS PROCEDURES:

**13A.6.1 General:** The nonlinear procedures provided in Sec. 13A.6 supplement the nonlinear procedures of Sec. 5.7 and 5.8 to accommodate the use of damping systems.

The stiffness and damping properties of the damping devices used in the models shall be based on or verified by testing of the damping devices as specified in Sec. 13A.10.

**13A.6.2. Nonlinear Static Analysis:** The nonlinear modeling described in Sec. 5.7.1 and the lateral loads described in Sec. 5.7.2 shall be applied to the seismic-force-resisting system. The resulting force-displacement curve shall be used in lieu of the assumed effective yield displacement, $D_y$, of Equation 13A.3.4-3 to calculate the effective ductility demand due to the design earthquake, $\mu_D$, and due to the maximum considered earthquake, $\mu_M$, in Eq. 13A.3.4-1 and 13A.3.4-2. The value of $(R/C_d)$ shall be taken as 1.0 in Equations 13A.4.3.4-1, 13A.4.3.4-2 and 13A.4.3.8-1 for the equivalent lateral force analysis procedure, and in Eq. 13A.5.3.4-1, 13A.5.3.4-2, 13A.5.3.6-1, and 13A.5.3.6-2 of the response spectrum analysis procedure.

**13A.6.3 Nonlinear Response History Analysis:** A nonlinear response history (time history) analysis shall utilize a mathematical model of the structure and the damping system as provided in Sec. 5.8 and this section. The model shall directly account for the nonlinear hysteretic behavior of elements of the structure and the damping devices to determine its response, through methods of numerical integration, to suites of ground motions compatible with the design response spectrum for the site.

The analysis shall be performed in accordance with Sec. 5.8 together with the requirements of this section.

**13A.6.3.1 Damping Device Modeling:** Mathematical models of displacement-dependent damping devices shall include the hysteretic behavior of the devices consistent with test data and accounting for all significant changes in strength, stiffness, and hysteretic loop shape.

Mathematical models of velocity-dependent damping devices shall include the velocity coefficient consistent with test data. If this coefficient changes with time and/or temperature, such behavior shall be modeled explicitly. The elements of damping devices connecting damper units to the structure shall be included in the model.
Exception: If the properties of the damping devices are expected to change during the duration of the time history analysis, the dynamic response may be enveloped by the upper and lower limits of device properties. All these limit cases for variable device properties must satisfy the same conditions as if the time dependent behavior of the devices were explicitly modeled.

13A.6.3.2 Response Parameters: In addition to the response parameters given in Sec. 5.8.3, the design earthquake and maximum considered earthquake displacements, velocities, and forces of the damping devices shall be determined.

13A.7 SEISMIC LOAD CONDITIONS AND ACCEPTANCE CRITERIA:

13A.7.1 General: Design forces and displacements determined in accordance with the equivalent lateral force analysis procedures of Sec. 13A.4 or the response spectrum analysis procedure of Sec. 13.5 shall be checked using the strength design criteria of these Provisions and the seismic loading conditions of the following sections.

13A.7.2 Seismic-Force-Resisting System: The seismic-force-resisting system shall meet the design provisions of Sec. 5.2.2 using seismic base shear and design forces determined in accordance with Sec. 13A.4.3 or Sec 13A.5.3.

The design earthquake story drift, \( \Delta_p \), as determined in either Sec. 13A.4.4.4 or 13A.5.4.4 shall not exceed \((R/C_d)\) times the allowable story drift, as obtained from Table 5.2.8, considering the effects of torsion as required in Sec. 5.2.8.

13A.7.3 Damping System: The damping system shall meet the provisions of Sec. 5.2.2 for seismic design forces determined in accordance with Sec. 13A.7.3.1 and the seismic loading conditions of Sec. 13A.7.3.2 and Sec. 13A.5.4.

13A.7.3.1 Modal Damping System Design Forces: Modal damping system design forces shall be calculated on the basis of the type of damping devices, and the modal design story displacements and modal design story velocities determined in accordance with either Sec. 13A.4.4 or Sec. 13A.5.4.

Exception: Modal design story displacements and velocities determined in accordance with either Sec. 13A.4.4 or Sec. 13A.5.4 shall be increased as required to envelop total design story displacements and velocities determined in accordance with Sec. 13A.6, when Sec. 13A.2.6.4.3 requires peak response to be confirmed by time history analysis.

1. Displacement-Dependent Damping Devices: Design seismic force in displacement-dependent damping devices shall be based on the maximum force in the device at displacements up to and including the design earthquake story drift, \( \Delta_p \).

2. Velocity-Dependent Damping Devices: Design seismic force in each mode of vibration of velocity-dependent damping devices shall be based on the maximum force in the device at velocities up to and including the design earthquake story velocity of the mode of interest.
Displacements and velocities used to determine design forces in *damping devices* at each story shall account for the angle of orientation from horizontal and consider the effects of increased floor response due to torsional motions.

**13A.7.3.2 Seismic Load Conditions and Combination of Modal Responses:** Seismic design force, $Q_E$, in each element of the *damping system* due to horizontal earthquake load shall be taken as the maximum force of the following three loading conditions:

1. **Stage of Maximum Displacement:** Seismic design force at the stage of maximum displacement shall be calculated in accordance with the following equation:

   $Q_E = Q_v \sqrt{\sum_m (Q_{mSFRS})^2} \pm Q_{DSD}$  \hspace{1cm} (13A.7.3.2-1)

   where:

   - $Q_{mSFRS}$ = Force in an element of the *damping system* equal to the design seismic force of the $m^{th}$ mode of vibration of the *seismic-force-resisting system* in the direction of interest.
   - $Q_{DSD}$ = Force in an element of the *damping system* required to resist design seismic forces of *displacement-dependent damping devices*.

   Seismic forces in elements of the *damping system*, $Q_{DSD}$, shall be calculated by imposing design forces of *displacement-dependent damping devices* on the *damping system* as pseudo-static forces. Design seismic forces of *displacement-dependent damping devices* shall be applied in both positive and negative directions at peak displacement of the *structure*.

2. **Stage of Maximum Velocity:** Seismic design force at the stage of maximum velocity shall be calculated in accordance with the following equation:

   $Q_E = \sqrt{\sum_m (Q_{mDSV})^2}$  \hspace{1cm} (13A.7.3.2-2)

   where:

   - $Q_{mDSV}$ = Force in an element of the *damping system* required to resist design seismic forces of *velocity-dependent damping devices* due to the $m^{th}$ mode of vibration of *structure* in the direction of interest.

   Modal seismic design forces in elements of the *damping system*, $Q_{mDSV}$, shall be calculated by imposing modal design forces of *velocity-dependent devices* on the non-deformed *damping system* as pseudo-static forces. Modal seismic design forces shall be applied in directions consistent with the deformed shape of the mode of interest. Horizontal restraint forces shall be applied at each floor Level $i$ of the non-deformed *damping system* concurrent with the design forces in *velocity-dependent damping devices* such that the horizontal displacement at each level of the *structure* is zero. At each floor Level $i$, restraint forces shall be proportional to and applied at the location of each mass point.
3. Stage of Maximum Acceleration: Seismic design force at the stage of maximum acceleration shall be calculated in accordance with the following equation:

\[ Q_E = \sqrt{\sum_m \left( C_{mFD} \Omega_m Q_{mSFRS} + C_{mFV} Q_{mDSV} \right)^2 + Q_{DSD}} \]  

(13A.7.3.2-3)

The force coefficients, \( C_{mFD} \) and \( C_{mFV} \), shall be determined from Tables 13A.7.3.2.1 and 13A.7.3.2.2, respectively, using values of effective damping determined in accordance with the following requirements:

a. For fundamental-mode response \((m = 1)\) in the direction of interest, the coefficients, \( C_{1FD} \) and \( C_{1FV} \), shall be based on the velocity power term, \( \alpha \), that relates device force to damping device velocity. The effective fundamental-mode damping, shall be taken as equal to the total effective damping of the fundamental mode less the hysteretic component of damping (e.g., \( \beta_{1D} - \beta_{HD} \)) at the response level of interest (i.e., \( \mu = \mu_D \) or \( \mu = \mu_M \)).

b. For higher-mode \((m > 1)\) or residual-mode response in the direction of interest, the coefficients, \( C_{mFD} \) and \( C_{mFV} \), shall be based on a value of \( \alpha \) equal to 1.0. The effective modal damping shall be taken as equal to the total effective damping of the mode of interest (e.g., \( \beta_{mD} \)). For determination of the coefficient \( C_{mFD} \), the ductility demand shall be taken as equal to that of the fundamental mode (e.g., \( \mu = \mu_D \)).

<table>
<thead>
<tr>
<th>Effective Damping</th>
<th>( \alpha \leq 0.25 )</th>
<th>( \alpha = 0.5 )</th>
<th>( \alpha = 0.75 )</th>
<th>( \alpha \geq 1.0 )</th>
<th>( C_{mFD} = 1.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \leq 0.05 )</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>( \mu \geq 1.0 )</td>
</tr>
<tr>
<td>0.1</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>( \mu \geq 1.0 )</td>
</tr>
<tr>
<td>0.2</td>
<td>1.00</td>
<td>0.95</td>
<td>0.94</td>
<td>0.93</td>
<td>( \mu \geq 1.1 )</td>
</tr>
<tr>
<td>0.3</td>
<td>1.00</td>
<td>0.92</td>
<td>0.88</td>
<td>0.86</td>
<td>( \mu \geq 1.2 )</td>
</tr>
<tr>
<td>0.4</td>
<td>1.00</td>
<td>0.88</td>
<td>0.81</td>
<td>0.78</td>
<td>( \mu \geq 1.3 )</td>
</tr>
<tr>
<td>0.5</td>
<td>1.00</td>
<td>0.84</td>
<td>0.73</td>
<td>0.71</td>
<td>( \mu \geq 1.4 )</td>
</tr>
<tr>
<td>0.6</td>
<td>1.00</td>
<td>0.79</td>
<td>0.64</td>
<td>0.64</td>
<td>( \mu \geq 1.6 )</td>
</tr>
<tr>
<td>0.7</td>
<td>1.00</td>
<td>0.75</td>
<td>0.55</td>
<td>0.58</td>
<td>( \mu \geq 1.7 )</td>
</tr>
<tr>
<td>0.8</td>
<td>1.00</td>
<td>0.70</td>
<td>0.50</td>
<td>0.53</td>
<td>( \mu \geq 1.9 )</td>
</tr>
<tr>
<td>0.9</td>
<td>1.00</td>
<td>0.66</td>
<td>0.50</td>
<td>0.50</td>
<td>( \mu \geq 2.1 )</td>
</tr>
<tr>
<td>( \geq 1.0 )</td>
<td>1.00</td>
<td>0.62</td>
<td>0.50</td>
<td>0.50</td>
<td>( \mu \geq 2.2 )</td>
</tr>
</tbody>
</table>

Table 13A.7.3.2.1 Force Coefficient, \( C_{mFD} \)\(^{a,b} \)
Unless analysis or test data support other values, the force coefficient $C_{mFD}$ for visco-elastic systems shall be taken as 1.0.

Interpolation shall be used for intermediate values of effective damping, $\alpha$, and $\mu$.

$C_{mFD}$ shall be taken as equal to 1.0 for values of $\mu$ greater than or equal to the values shown.

### Table 13A.7.3.2.2 Force Coefficient, $C_{mFV}$

<table>
<thead>
<tr>
<th>Effective Damping</th>
<th>$\alpha \leq 0.25$</th>
<th>$\alpha = 0.5$</th>
<th>$\alpha = 0.75$</th>
<th>$\alpha \geq 1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\leq 0.05$</td>
<td>1.00</td>
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<td>0.10</td>
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<tr>
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</tr>
<tr>
<td>0.2</td>
<td>1.00</td>
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<td>0.46</td>
<td>0.37</td>
</tr>
<tr>
<td>0.3</td>
<td>1.00</td>
<td>0.64</td>
<td>0.58</td>
<td>0.51</td>
</tr>
<tr>
<td>0.4</td>
<td>1.00</td>
<td>0.70</td>
<td>0.69</td>
<td>0.62</td>
</tr>
<tr>
<td>0.5</td>
<td>1.00</td>
<td>0.75</td>
<td>0.77</td>
<td>0.71</td>
</tr>
<tr>
<td>0.6</td>
<td>1.00</td>
<td>0.80</td>
<td>0.84</td>
<td>0.77</td>
</tr>
<tr>
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<td>0.83</td>
<td>0.90</td>
<td>0.81</td>
</tr>
<tr>
<td>0.8</td>
<td>1.00</td>
<td>0.90</td>
<td>0.94</td>
<td>0.90</td>
</tr>
<tr>
<td>0.9</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\geq 1.0$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

### 13A.7.3.3 Combination of Load Effects:

The effects on the damping system and its components due to gravity loads and seismic forces shall be combined in accordance with Sec. 5.2.7 using the effect of horizontal seismic forces, $Q_E$, determined in accordance with Sec. 13A.7.3.2.

**Exception:** The reliability factor, $\rho$, shall be taken as equal to 1.0 in all cases and the special load combinations of Sec. 5.2.7.1 need not apply to the design of the damping system.

### 13A.7.3.4 Inelastic Response Limits:

Elements of the damping system may exceed strength limits for design loads provided it is shown by analysis or test that:
1. Inelastic response does not adversely affect damping system function.

2. Element forces calculated in accordance with Sec. 13A.7.3.2, using a value of $\Omega_o$, taken as equal to 1.0, do not exceed the strength required to meet the load combinations of Sec. 5.2.7.

13A.8 DETAILED SYSTEM REQUIREMENTS:

13A.8.1 Damping Device Design: The design, construction and installation of damping devices shall be based on maximum earthquake response and the following load conditions:

1. Low-cycle, large displacement degradation due to seismic loads;
2. High-cycle, small-displacement degradation due to wind, thermal or other cyclic loads;
3. Forces or displacements due to gravity loads;
4. Adhesion of device parts due to corrosion or abrasion, biodegradation, moisture or chemical exposure; and
5. Exposure to environmental conditions, including but not limited to temperature, humidity, moisture, radiation (e.g., ultraviolet light) and reactive or corrosive substances (e.g., salt water).

Damping devices subject to failure by low-cycle fatigue shall resist wind forces without slip, movement, or inelastic cycling.

The design of damping device shall incorporate the range of thermal conditions, device wear, manufacturing tolerances, and other effects that cause device properties to vary during the lifetime of the device.

13A.8.2 Multi-Axis Movement: Connection points of damping devices shall provide sufficient articulation to accommodate simultaneous longitudinal, lateral, and vertical displacements of the damping system.

13A.8.3 Inspection and Periodic Testing: Means of access for inspection and removal of all damping devices shall be provided.

The registered design professional responsible for design of the structure shall establish an appropriate inspection and testing schedule for each type of damping device to ensure that the devices respond in a dependable manner throughout device design life. The degree of inspection and testing shall reflect the established in-service history of the damping devices, and the likelihood of change in properties over the design life of devices.

13A.8.4 Manufacturing Quality Control: The registered design professional responsible for design of the structure shall establish a quality control plan for the manufacture of damping devices. As a minimum, this plan shall include the testing requirements of Sec. 13A.10.3.

13A.9 DESIGN REVIEW:

13A.9.1 General: Review of the design of the damping system and related test programs shall be performed by an independent engineering panel including persons licensed in the appropriate disciplines and experienced in seismic analysis including the theory and application of energy dissipation methods.
13A.9.2 **Review Scope:** The design review shall include the following:

1. Review of the earthquake ground motions used for design.
2. Review of design parameters of *damping devices*, including device test requirements, device manufacturing quality control and assurance, and scheduled maintenance and inspection requirements.
3. Review of nonlinear analysis methods incorporating the requirements of Sec. 5.8.4.
4. Review of the preliminary design of the *seismic-force-resisting system* and the *damping system*.
5. Review of the final design of the *seismic-force-resisting system* and the *damping system* and all supporting analyses.

**13A.10 REQUIRED TESTS OF DAMPING DEVICES:**

13A.10.1 **General:** The force-velocity-displacement and damping properties used for the design of the *damping system* shall be based on the prototype tests of a selected number of *damping devices*, as specified in Sec. 13A.10.2.1. 

The fabrication and quality control procedures used for all prototype and production *damping devices* shall be identical.

**13A.10.2 Prototype Tests:**

13A.10.2.1 **General:** The following tests shall be performed separately on two full-size *damping devices* of each type and size used in the design, in the order listed below.

Representative sizes of each type of device may be used for prototype testing, provided both of the following conditions are met:

(1) Fabrication and quality control procedures are identical for each type and size of devices used in the *structure*.

(2) Prototype testing of representative sizes is accepted by the *registered design professional* responsible for design of the *structure*.

Test specimens shall not be used for construction, unless they are accepted by the *registered design professional* responsible for design of the *structure* and meet the requirements of Sec. 13A.10.2 and Sec. 13A.10.3.

13A.10.2.2 **Data Recording:** The force-deflection relationship for each cycle of each test shall be recorded.

13A.10.2.3 **Sequence and Cycles of Testing:** For the following test sequences, each *damping device* shall be subjected to gravity load effects and thermal environments representative of the installed condition. For seismic testing, the displacement in the devices calculated for the
maximum considered earthquake, termed herein as the maximum earthquake device
displacement, shall be used.

1. Each damping device shall be subjected to the number of cycles expected in the design
windstorm, but not less than 2000 continuous fully reversed cycles of wind load. Wind load
shall be at amplitudes expected in the design wind storm, and applied at a frequency equal to
the inverse of the fundamental period of the building \( f_i = 1/T_i \).

   **Exception:** Damping devices need not be subjected to these tests if they are not
subject to wind-induced forces or displacements, or if the design wind force is less
than the device yield or slip force.

2. Each damping device shall be loaded with 5 fully reversed, sinusoidal cycles at the maximum
earthquake device displacement at a frequency equal to \( 1/T_{im} \) as calculated in Sec. 13A.4.3.5.
Where the damping device characteristics vary with operating temperature, these tests shall
be conducted at a minimum of 3 temperatures (minimum, ambient, and maximum) that
bracket the range of operating temperatures.

   **Exceptions:** Damping devices may be tested by alternative methods provided each
of the following conditions is met:
   a. Alternative methods of testing are equivalent to the cyclic testing requirements of
this section.
   b. Alternative methods capture the dependence of the damping device response on
ambient temperature, frequency of loading, and temperature rise during testing.
   c. Alternative methods are accepted by the registered design professional
responsible for the design of the structure.

3. If the force-deformation properties of the damping device at any displacement less than or
equal the maximum earthquake device displacement change by more than 15 percent for
changes in testing frequency from \( 1/T_{im} \) to \( 2.5/T_i \), then the preceding tests shall also be
performed at frequencies equal to \( 1/T_i \) and \( 2.5/T_i \).

   If reduced-scale prototypes are used to qualify the rate dependent properties of damping
devices, the reduced-scale prototypes should be of the same type and materials, and
manufactured with the same processes and quality control procedures, as full-scale
prototypes, and tested at a similitude-scaled frequency that represents the full-scale loading
rates.

**13A.10.2.4 Testing Similar Devices:** Damping devices need not be prototype tested provided
that both of the following conditions are met:

1. The damping device manufacturer substantiates the similarity of previously tested devices.
2. All pertinent testing and other damping device data are made available to, and accepted by
the registered design professional responsible for the design of the structure.
13A.10.2.5 Determination of Force-Velocity-Displacement Characteristics: The force-velocity displacement characteristics of a damping device shall be based on the cyclic load and displacement tests of prototype devices specified above. Effective stiffness of a damping device shall be calculated for each cycle of deformation using Eq. 13.9.3-1.

13A.10.2.6 Device Adequacy: The performance of a prototype damping device shall be assessed as adequate if all of the conditions listed below are satisfied. The 15-percent limits specified below may be increased by the registered design professional responsible for the design of the structure provided that the increased limit has been demonstrated by analysis to not have a deleterious effect on the response of the structure.

13A.10.2.6.1 Displacement-Dependent Devices:

1. For Sec. 13A.10.2.3 Test 1, no signs of damage including leakage, yielding, or breakage.

2. For Sec. 13A.10.2.3 Tests 2 and 3, the maximum force and minimum force at zero displacement for a damping device for any one cycle does not differ by more than plus or minus 15 percent from the average maximum and minimum forces at zero displacement as calculated from all cycles in that test at a specific frequency and temperature.

3. For Sec. 13A.10.2.3 Tests 2 and 3, the maximum force and minimum force at maximum earthquake device displacement for a damping device for any one cycle does not differ by more than plus or minus 15 percent from the average maximum and minimum forces at the maximum earthquake device displacement as calculated from all cycles in that test at a specific frequency and temperature.

4. For Sec. 13A.10.2.3 Tests 2 and 3, the area of hysteresis loop \((E_{loop})\) of a damping device for any one cycle does not differ by more than plus or minus 15 percent from the average area of the hysteresis loop as calculated from all cycles in that test at a specific frequency and temperature.

5. The average maximum and minimum forces at zero displacement and maximum earthquake displacement, and the average area of the hysteresis loop \((E_{loop})\), calculated for each test in the sequence of Sec. 13A.10.2.3 Tests 2 and 3, shall not differ by more than plus or minus 15 percent from the target values specified by the registered design professional responsible for the design of the structure.

13A.10.2.6.2 Velocity-Dependent Devices:

1. For Sec. 13A.10.2.3 Test 1, no signs of damage including leakage, yielding, or breakage.

2. For velocity-dependent damping devices with stiffness, the effective stiffness of a damping device in any one cycle of Tests 2 and 3 of Sec. 13A.10.2.3 does not differ by more than plus or minus 15 percent from the average effective stiffness as calculated from all cycles in that test at a specific frequency and temperature.
3. For Sec. 13A.10.2.3 Tests 2 and 3, the maximum force and minimum force at zero displacement for a damping device for any one cycle does not differ by more than plus or minus 15 percent from the average maximum and minimum forces at zero displacement as calculated from all cycles in that test at a specific frequency and temperature.

4. For Sec. 13A.10.2.3 Tests 2 and 3, the area of hysteresis loop (E_loop) of a damping device for any one cycle does not differ by more than plus or minus 15 percent from the average area of the hysteresis loop as calculated from all cycles in that test at a specific frequency and temperature.

5. The average maximum and minimum forces at zero displacement, effective stiffness (for damping devices with stiffness only), and average area of the hysteresis loop (E_loop) calculated for each test in the sequence of Sec. 13A.10.2.3 Tests 2 and 3, shall not differ by more than plus or minus 15 percent from the target values specified by the registered design professional responsible for the design of the structure.

**13A.10.3 Production Testing:** Prior to installation in a building, damping devices shall be tested to ensure that their force-velocity-displacement characteristics fall within the limits set by the registered design professional responsible for the design of the structure. The scope and frequency of the production-testing program shall be determined by the registered design professional responsible for the design of the structure.